kd education academy (9582701166

Time : 10 Hour

STD 11 Maths

kd90+ ch- 6 permutations and combinations

Total Marks : 330

*** Choose the right answer from the given options. [1 Marks Each]** **[112]**

1. Find the number of rectangles and squares in an 8 by 8 chess board respectively.

   (A) 296, 204 (B) 1092, 204 (C) 204, 1092 (D) 204, 1296

   Ans. :

   b. 1092, 204

   Solution:

   Chess board consists of 9 horizontal 9 vertical lines. A rectangle can be formed by any two horizontal and two vertical lines.

   Number of rectangles $= {}^9\mathrm{C}_2 \times {}^9\mathrm{C}_2 = 1296$. For squares there is one 8 by 8 square four 7 by 7 squares, nine 6 by 6 squares and like this

   Number of squares on chess board $= 1^2 + 2^2 \ldots\ldots 8^2 = 204$

   Only rectangles $= 1296 - 204 = 1092$

2. The number of ways in which 6 men add 5 women can dine at a round table, if no two women are to sit together, is given by:

   (A) 30 (B) 5! × 5! (C) 5! × 4! (D) 7! × 5!

   Ans. :

   b. 5! × 5!

   Solution:

   Again, 6 girls can be arranged among themselves in 5! ways in a circle.

   So, the number of arrangements where boys and girls sit attentively in a circle = 5! × 5!

3. There are 6 letters and 6 directed envelopes.Find the number of ways in which all letters are put in the wrong envelopes.

   (A) 260 (B) 265 (C) 270 (D) 275

   Ans. :

   b. 265

4. The number of different ways in which 8 persons can stand in a row so that between two particular persons A and B there are always two persons, is:

   (A) 60 × 5! (B) 15 × 4! × 5! (C) 4! × 5! (D) None of these.

   Ans. :

   a. 60 × 5!

   Solutions:

The four people, i.e A, B and the two persons between them are always together. Thus, they can be considered as a single person. So, along with the remaining 4 persons, there are now total 5 people who need to be arranged. This can be done in 5! ways. But, the two persons that have to be included between A and B could be selected out of the remaining 6 people in $^6P_2$ ways, which is equal to 30. For each selection, these two persons standing between A and B can be arranged among themselves in 2 ways.

∴ Total number of arrangements = 5! × 30 × 2 = 60 × 5!

5. The number of ways in which four particular persons A, B, C, D, and six more persons can stand in a queue so that A always stands before B B, before C and C before D, is:

(A) $7!4!$ (B) $10! - 7!4!$ (C) $\frac{10!}{4!}$ (D) None of these

Ans. :

c. $\frac{10!}{4!}$

6. The number of ways to arrange the letters of the word CHEESE are:

(A) 120 (B) 240 (C) 720 (D) 6

Ans. :

a. 120

Solutions:

Total number of arrangements of the letters of the word CHEESE = Number of arrangements of 6 things taken all at a time, of which 3 are of one kind $= \frac{6!}{3!} = 120$

7. Seven different lecturers are to deliver lectures in seven periods of a class on a particular day. A B, and C are three of the lecturers.The umber of ways in which a routine for the day can be made such that A delivers his lecture before B and B before C, is:

(A) 420 (B) 120 (C) 210 (D) 840

Ans. :

d. 840

8. The greatest number that can be formed by the digits 7, 0, 9, 8, 6, 3

(A) 9, 87, 360 (B) 9, 87, 063 (C) 9, 87, 630 (D) 9, 87, 603

Ans. :

c. 9, 87, 630

Solution:

The greatest number that can be formed by the digits 7, 0, 9, 8, 6, 3 is 9 8 7 6 3 0 To achieve this arrange the given numbers in descending order.

9. If $^{20}C_r = {}^{20}C_{r\text{-}10}$ is then $^{18}C_r$ equal to:

(A) 4896 (B) 816 (C) 1632 (D) None of these.

Ans. :

b. 816

Solution:

$r + r - 10 = 20$

$\Rightarrow 2r - 10 = 20$

$\Rightarrow 2r = 30$

$\Rightarrow r = 15$

Now,

${}^{18}C_r = {}^{18}C_{15}$

$\therefore\ {}^{18}C_{15} = {}^{18}C_3$

$\therefore\ {}^{18}C_3 = \frac{18}{3} \times \frac{17}{2} \times 16$

$= 816$

10. The number of words that can be formed out of the letters of the word "ARTICLE" so that vowels occupy even places is:

(A) 574 (B) 36 (C) 754 (D) 144

Ans. :

d. 144

Solutions:

The word ARTICLE consists of 3 vowels that have to be arranged in the three even places. This can be done in 3! ways. And, the remaining 4 consonants can be arranged among themselves in 4! ways.

$\therefore$ Total number of ways = 3! × 4! = 144

11. The number of permutations of n different things taking r at a time when 3 particular things are to be included is:

(A) ${}^{n-3}P_{r-3}$ (B) ${}^{n-3}P_r$ (C) ${}^{n}P_{r-3}$ (D) $r!\ {}^{n-3}C_{r-3}$

Ans. :

d. $r!\ {}^{n-3}C_{r-3}$

Solutions:

Here, we have to permute n things of which 3 things are to be included. So, only the remaining (n − 3) things are left for permutation, taking (r − 3) things at a time. This is because 3 things have already been included. But, these r things can be arranged in r! ways.

$\therefore$ Total number of permutations $= r!\ {}^{n-3}C_{r-3}$

12. The number of ways in which 10 different diamonds can be arranged to form a necklace, is:

(A) 181440 (B) 161400 (C) 261960 (D) None of these

Ans. :

a. 181440

13. A garrison of nn men had enough food to last for 30 days. After 10 days, 50 more men joined them. If the food now lasted for 1616 days, what is the value of n?

(A) 200 (B) 240 (C) 280 (D) 320

Ans. :

a. 200

Solution:

After 10 days, the food for n men is there for 20 days.

This food can be eaten by $(\mathrm{n}+50)$ men in 16 days.

$\therefore 20\mathrm{n}=16(\mathrm{n}+5)$

$\therefore \mathrm{n}=200$

14. If in a group of n distinct objects, the number of arrangements of 4 objects is 12 times the number of arrangements of 2 objects, then the number of objects is:

(A) 10 (B) 8 (C) 6 (D) None of these

Ans. :

c. 6

Solutions:

According to the question:

${}^{\mathrm{n}}\mathrm{P}_4=12\times {}^{\mathrm{n}}\mathrm{P}_2$

$\Rightarrow \frac{\mathrm{n}!}{(\mathrm{n}-4)!}=12\times\frac{\mathrm{n}!}{(\mathrm{n}\text{-}2)!}$

$\Rightarrow \frac{(\mathrm{n}-2)!}{(\mathrm{n}-4)!}=12$

$\Rightarrow (\mathrm{n}-2)(\mathrm{n}-3)=4\times 3$

$\Rightarrow \mathrm{n}-2=4$

$\Rightarrow \mathrm{n}=6$

15. If the letters of the word KRISNA are arranged in all possible ways and these words are written out as in a dictionary, then the rank of the word KRISNA is:

(A) 324 (B) 341 (C) 359 (D) None of these

Ans. :

a. 324

Solutions:

When arranged alphabetically, the letters of the word KRISNA are A, I, K, N, R and S.

Number of words that will be formed with A as the first letter = Number of arrangements of the remaining 5 letters = 5!

Number of words that will be formed with I as the first letter = Number of arrangements of the remaining 5 letters = 5!

$\therefore$ The number of words beginning with KA = Number of arrangements of the remaining 4 letters = 4!

The number of words beginning with KA = Number of arrangements of the remaining 4 letters = 4!

The number of words starting with KN = Number of arrangements of the remaining 4 letters = 4!

Alphabetically, the next letter will be KR.

Number of words starting with KR followed by A, i.e. KRA = Number of arrangements of the remaining 3 letters = 3!

Number of words starting with KRI followed by A, i.e. KRIA = Number of arrangements of the remaining 2 letters = 2!

Number of words starting with KRI followed by N, i.e. KRIN = Number of arrangements of the remaining 2 letters = 2!

The first word beginning with KRIS is the word KRISAN and the next word is KRISNA.

$\therefore$ Rank of the word KRISNA = 5! + 5! + 4! + 4! + 4! + 3! + 2! + 2! + 2

= 324

16. The total number of 9 digit numbers of different digits is:

(A) 99! (B) 9! (C) 8 × 9! (D) 9 × 9!

Ans. :

d. 9 × 9!

Solution:

Given digit in the number = 9

1st place can be filled = 9 ways = 9 (from 1 - 9 any number can be placed at first position)

2nd place can be filled = 9 ways (from 0 - 9 any number can be placed except the number which is placed at the first position)

3rd place can be filled = 8 ways.

4th place can be filled = 7 ways.

5th place can be filled = 6 ways.

6th place can be filled = 5 ways.

7th place can be filled = 4 ways.

8th place can be filled = 3 ways.

9th place can be filled = 2 ways.

So total number of ways = 9 × 9 × 8 × 7 × 6 × 5 × 4 × 3 × 2

= 9 × 9!

17. Permutation relates to the act of arranging all the members of a set into some sequence or order.

(A) True (B) False

(C) Can be true or false. (D) Can not say.

Ans. :

a. True

Solution:

True, permutation relates to the act of arranging all the members of a set into some sequence or order.

18. Amy and Adam are making boxes of truffles to give out as wedding favors. They have an unlimited supply of 5 different types of truffles. If each box holds 2 truffles of different types, how many different boxes can they make?

(A) 12 (B) 10 (C) 15 (D) 20

Ans. :

b. 10

Solution:

10 boxes In every combination, 2 types of truffles will be in the box, and 3 types of truffles will not.

Therefore, this problem is a question about the number of anagrams that can be made from the "word" YYNNN:

$\frac{5!}{2!3!} = \frac{5\times4\times3\times2\times1}{3\times2\times1\times2\times1} = 5 \times 2 = 10$

19. If ${}^{5}\mathrm{P}_{\mathrm{r}} = {}^{26}\mathrm{p}_{\mathrm{r}} - 1$, then the value of r is:

(A) 10 (B) 3 (C) 0 (D) None of these

Ans. :

a. 3

20. Choose the correct answer.

The number of ways in which we can choose a committee from four men and six women so that the committee includes at least two men and exactly twice as many women as men is.

(A) 94 (B) 126 (C) 128 (D) None

Ans. :

a. 94

Solution:

Number of men = 4

Number of women = 6

We are given that the committee includes 2 men and exactly twice as many women as men.

Thus, the possible selection can be 2 men and 4 women and 3 men and 6 women

So, the number of committee = ${}^{4}C_{2} \times {}^{6}C_{4} + {}^{4}C_{3} \times {}^{6}C_{6}$ = 6 × 15 + 4 1 = 90 + 4 = 94

21. Let Tn denote the number of triangles which can be formed using the vertices of a regular polygon of n sides.If $\mathrm{T_{n+1} - T_n = 21}$, then n equals:

(A) 5 (B) 7 (C) 6 (D) 4

Ans. :

b. 7

22. If $^{15}\mathrm{C}_{3\mathrm{r}} = {}^{15}\mathrm{C}_{\mathrm{r}+3}$, is then equal to:

(A) 5 (B) 4 (C) 3 (D) 2

Ans. :

c. 3

Solution:

$3\mathrm{r} + \mathrm{r} + 3 = 15$

$\Rightarrow 4\mathrm{r} + 3 = 15$

$\Rightarrow 4\mathrm{r} = 12$

$\Rightarrow \mathrm{r} = 3$

23. Three persons enter a railway compartment. If there are 5 seats vacant, in how many ways can they take these seats?

(A) 60 (B) 20 (C) 15 (D) 125

Ans. :

a. 60

Solution:

Three persons can take 5 seats in $^5C_3$ ways. Moreover 3 persons can sit in 3! ways.

Required number of ways $^5\mathrm{C}_3 \times 3!$

$= 10 \times 6 = 60$

24. How many 5 - digit telephone numbers can be constructed using the digits 0 to 9, if each number starts with 67 and no digit appears more than once:

(A) 336 (B) 337 (C) 335 (D) None of these

Ans. :

a. 336

25. How many ways can 6 coins be chosen from 20, one rupees coins, 10 fifty paise coins, 7 twenty paise coins:

(A) 28 (B) 56 (C) $^{37}\mathrm{C}_6$ (D) 38

Ans. :

a. 28

26. The value of $({}^7\mathrm{C}_0 + {}^7\mathrm{C}_1) + ({}^7\mathrm{C}_1 + {}^7\mathrm{C}_3) + \ldots\ldots + ({}^7\mathrm{C}_6 + {}^7\mathrm{C}_7)$ is:

(A) $2^7 - 1$ (B) $2^8 - 2$ (C) $2^8 - 1$ (D) 2^8

Ans. :

b. $2^8 - 2$

Solution:

$({}^7C_0 + {}^7C_1) + ({}^7C_1 + {}^7C_3) + ({}^7C_2 + {}^7C_3) + ({}^7C_3 + {}^7C_4) + \ldots\ldots$

$= 1 + 2 \times {}^7C_1 + 2 \times {}^7C_2 + 2 \times {}^7C_3 + 2 \times {}^7C_4 + 2 \times {}^7C_5 ..$

$= 2 + 2^2({}^7C_1 + {}^7C_2 + {}^7C_3)$

$= 2 + 2^2(7 + \frac{7}{2} \times 6 + \frac{7}{3} \times \frac{6}{2} \times 5)$

$= 2 + 252$

$= 254$

$= 2^8 - 2$

27. The number of ways in which 6 men can be arranged in a row so that three particular men are consecutive, is:

(A) 4! × 3! (B) 4! (C) 3! × 3! (D) None of these

Ans. :

a. 4! × 3!

Solutions:

According to the question, 3 men have to be 'consecutive' means that they have to be considered as a single man. But, these 3 men can be arranged among themselves in 3! ways. And, the remaining 3 men, along with this group, can be arranged among themselves in 4! ways.

$\therefore$ Total number of arrangements = 4! × 3!

28. From a committee of 8 persons, in how many ways can we choose a chairman and a vice - chairman assuming one person cannot hold more than one position:

(A) 54 (B) 55 (C) 52 (D) 56

Ans. :

d. 56

29. Factorial of negative numbers is always greater than 1:

(A) True (B) False (C) Either (D) Neither

Ans. :

b. False

Solution:

Factorial: product of an integer with integer less than it.

Factorial can be interpolated using gamma function and gamma function and gamma function is not defined for negative integer.

Factorial is not defined for negative integer.

30. If ${}^{n+1}C_3 = 2\ {}^nC_2$, then the value of n is:

(A) 3 (B) 4 (C) 5 (D) 6

Ans. :

d. 6

Solution:

Given, ${}^{n+1}C_3 = 2\,{}^{n}C_2,$

$\Rightarrow \left[\frac{(n+1)!}{(n+1-3)} \times 3!\right] = \frac{2n!}{(n-2)} \times 2!$

$\Rightarrow \left[\frac{(n\times 1!)}{(n-2)} \times 3!\right] = \frac{2n!}{(n-2)} \times 2$

$\Rightarrow \frac{n}{3!} = 1$

$\Rightarrow \frac{n}{6} = 1$

$\Rightarrow n = 6$

31. If ${}^{20}C_r = {}^{20}C_{r+4}$ is then ${}^{r}C_3$ equal to:

(A) 54 (B) 56 (C) 58 (D) none of these.

Ans. :

b. 56

Solution:

$r + r + 4 = 20$

$\Rightarrow 2r + 4 = 20$

$\Rightarrow 2r = 16$

$\Rightarrow r = 8$

Now,

${}^{r}C_3 = {}^{8}C_3$

$\therefore\ {}^{8}C_3 = {}^{8}C_3$

$\therefore\ {}^{8}C_3 = \frac{8!}{3!5!}$

$= \frac{8\times 7\times 6}{3\times 2\times 1} = 56$

32. Choose the correct answer.

The number of 5-digit telephone numbers having atleast one of their digits repeated is.

(A) 90,000 (B) 10,000 (C) 30,240 (D) 69,760

Ans. :

d. 69,760

Solusion:

Total number of telephone numbers when there is no restriction = 10^5 Also number of telephone numbers having all digits different = ${}^{l0}P_5$ Required number of ways = $10^5 - {}^{10}P_5$ = 1000000 − 10 × 9 × 8 × 7 × 6 = 1000000 - 30240 = 69760

33. The number of different signals which can be given from 6 flags of different colours taking one or more at a time, is:

(A) 1958 (B) 1956 (C) 16 (D) 64

Ans. :

b. 1956

Solutions:

Number of permutations of six signals taking 1 at a time = $^6P_1$

Number of permutations of six signals taking 2 at a time = $^6P_2$

Number of permutations of six signals taking 3 at a time = $^6P_3$

Number of permutations of six signals taking 4 at a time = $^6P_4$

Number of permutations of six signals taking 5 at a time = $^6P_5$

Number of permutations of six signals taking all at a time = $^6P_6$

$\therefore$ Total number of signals

$= \frac{6!}{5!} + \frac{6!}{4!} + \frac{6!}{3!} + \frac{6!}{2!} + \frac{6!}{1!} + 6!$
$= 6 + 30 + 120 + 360 + 720 + 720$
$= 1956$

34. The number of 6 - digit numbers can be formed from the digits 0, 1, 3, 5, 7 and 9 which are divisible by 10 and no digit is repeated are:

(A) 110 (B) 120 (C) 130 (D) 140

Ans. :

b. 120

Solution:

A number is divisible by 10 if the unit digit of the number is 0.
Given digits are 0, 1, 3, 5, 7, 9
Now we fix digit 0 at unit place of the number.
Remaining 5 digits can be arranged in 5! ways
So, total 6 - digit numbers which are divisible by 10 = 5! = 120

35. There are 4 parcels and 5 post offices.In how many ways can 4 parcels be got registered:

(A) 20 (B) 4^5 (C) 5^4 (D) $5^4 - 4^5$

Ans. :

c. 5^4

36. Arranging people, digits, numbers, alphabets, letters, and colours are example of:

(A) Combination (B) Permutation (C) Sets (D) Lists

Ans. :

b. Permutation

Solution:

Permutation : Arranging people, digits, numbers, alphabets, letters, and colours:

37. If ${}^{\text{n}}\text{P}_5 = 60\ {}^{\text{n-1}}\text{P}_3$, the value of n is:

(A) 6 (B) 10 (C) 12 (D) 16

Ans. :

b. 10

Solution:

Given that ${}^{\text{n}}\text{P}_5 = 60\ {}^{\text{n-1}}\text{P}_3$,

We know that $\text{P}(\text{n}, \text{r}) = {}^{\text{n}}\text{P}_r = \frac{\text{n!}}{\text{(n-r)!}}$

Now, apply the formula on both sides to get the value of n.

$\frac{\text{n!}}{(\text{n}-5)!} = 60\left[\frac{(\text{n}-1)!}{(\text{n}-1)-3!}\right]$

On solving the above equation, we get n= -6 and n=10.

Since the value of n cannot be negative, the value of n is 10.

38. ${}^{x}C_7 - {}^{x}C_5 = 0$, then x =:

(A) 7 (B) 5 (C) 12 (D) 10

Ans. :

c. 12

39. Out of 100 students 50 fail in English and 30 in Maths. If 12 students fail in both English and Maths, then the number of students passing both the subjects is:

(A) 26 (B) 28 (C) 30 (D) 32

Ans. :

d. 32

Solution:

Total number of students = 100

Number of students fail in English = (50 - 12) = 38

Number of students fail in Maths = (30 - 12) = 18

Number of students fail in both = 12

Therefore total failing students = (38 + 18 + 12) = 68

Pass in both the subjects = (100 - 68) = 32 students

32 students have passed in both subjects.

40. Six boys and six girls sit along a line alternately in x ways and along a circle (again alternatively in y ways), then:

(A) x = y (B) y = 12x (C) x = 10y (D) x = 12y

Ans. :

d. x = 12y

Solution:

Given, six boys and six girls sit along a line alternately in x ways and along a circle

(again alternatively in y ways).

Now, $\mathrm{x} = 6! \times 6! + 6! \times 6!$

$\Rightarrow \mathrm{x} = 2 \times (6!)^2$

and $\mathrm{y} = 5! \times 6!$

Now, $\frac{\mathrm{x}}{\mathrm{y}} = \frac{2 \times (6!)^2}{(5! \times 6!)}$

$\Rightarrow \frac{\mathrm{x}}{\mathrm{y}} = \frac{(2 \times 6! \times 6!)}{5! \times 6}$

$\Rightarrow \frac{\mathrm{x}}{\mathrm{y}} = \frac{(2 \times 6!)}{5!}$

$\Rightarrow \frac{\mathrm{x}}{\mathrm{y}} = \frac{(2 \times 6 \times 5!)}{5!}$

$\frac{\mathrm{x}}{\mathrm{y}} = 12$

$\Rightarrow \mathrm{x} = 12\mathrm{y}$

41. There are 15 points in a plane, no two of which are in a straight line except 4, all of which are in a straight line.The number of triangle that can be formed by using these 15 points is:

(A) ${}^{15}\mathrm{C}_3$ (B) 490 (C) 451 (D) 415

Ans. :

c. 451

Solution:

The required number of triangle $= {}^{15}\mathrm{C}_3 - {}^{4}\mathrm{C}_3 = 455 - 4 = 451$

42. An automobile dealer provides motorcycles and scooters in three body patterns and 4 different colors each. The number of choices open to a customer is:

(A) $5C_3$ (B) $4C_3$ (C) 4 × 3 (D) 4 × 3 × 2

Ans. :

d. 4 × 3 × 2

43. If a secretary and a joint secretary are to be selected from acommittee of 11 members, then in how many ways can they be selected:

(A) 110 (B) 55 (C) 22 (D) 11

Ans. :

b. 55

44. A group of 1200 persons consisting of captains and soldiers is travelling in a train. For every 15 soldiers there is one captain. The number of captains in the group is:

(A) 85 (B) 80 (C) 75 (D) 70

Ans. :

c. 75

Solution:

Out of 16 men, there is a captain.

Number of captains in 1200 men = 1200 ÷ 16 = 75.

45. Choose the correct answer.

Total number of words formed by 2 vowels and 3 consonants taken from 4 vowels and 5 consonants is equal to.

(A) 60 (B) 120 (C) 7200 (D) 72

Ans. :

c. 7200

Solution:

Given that total number of vowels = 4

and number of consonants = 5

The total of words formed by 2 vowels and 3 consonents

$= {}^4C_2 \times {}^5C_3 = \frac{4!}{2!\,2!} \times \frac{5!}{3!\,2!} = \frac{4\times3\times2!}{2\times1\times2!} \times \frac{5\times4\times3!}{3!\times2}$

Now permutation of 2 vowels and 3 consonants = 5! = 5 × 4 × 3 × 2 × 1 = 120

So, the total number of words = 60 × 120 = 7200.

46. Six identical coins are arranged in a row.The number of ways in which the number of tails is equal to the number of heads is:

(A) 20 (B) 9 (C) 120 (D) 40

Ans. :

a. 20

47. A 5-digit number divisible by 3 is to be formed using the digits 0, 1, 2, 3, 4 and 5 without repetition. The total number of ways in which this can be done is:

(A) 216 (B) 600 (C) 240 (D) 3125

Ans. :

a. 216

Solutions:

A number is divisible by 3 when the sum of the digits of the number is divisible by 3. Out of the given 6 digits, there are only two groups consisting of 5 digits whose sum is divisible by 3.

= 1 + 2 + 3 + 4 + 5 = 15

= 0 + 1 + 2 + 4 + 5 = 12

Using the digits 1, 2, 3, 4 and 5, the 5 digit numbers that can be formed = 5!

Similarly, using the digits 0, 1, 2, 4 and 5, the number that can be formed = 5! - 4! {since the first digit cannot be 0}

∴ Total numbers that are possible = 5! + 5! - 4! = 240 - 24 = 216

48. A car driver knows four different routes from Delhi to Amritsar. From Amritsar to Pathankot, he knows three different routes and from Pathankot to Jammu he knows two different routes. How many routes does he know from Delhi to Jammu?

(A) 4 (B) 8 (C) 12 (D) 24

Ans. :

d. 24

Solution:

The car driver can reach Amritsar in 4 ways.From each of these four ways, he can reach Pathankot in 3 different ways and so he can reach from Delhi to Pathankot in (4 × 3) i.e 12 ways. Again from Pathankot to Jammu, there are 2 ways. Hence he can reach Jammu from Delhi in (12 × 2) i.e. in 24 ways.

49. There are mn letters and n post boxes.The number of ways in which these letters can be posted is:

(A) $(\mathrm{mn})^{\mathrm{n}}$ (B) $(\mathrm{mn})^{\mathrm{m}}$ (C) m^{mn} (D) n^{mn}

Ans. :

d. n^{mn}

Solution:

Every letter can be posted in any of the n post boxes.

$\therefore$ Total number of ways $= \mathrm{n} \times \mathrm{n} \times \mathrm{n} \times \ldots . .(\mathrm{m} \times \mathrm{n}) = \mathrm{n}^{\mathrm{mn}}$

50. In how many ways a committee consisting of 5 men and 3 women, can be chosen from 9 men and 12 women:

(A) 10258 (B) 16870 (C) 27720 (D) 38982

Ans. :

c. 27720

51. Total number of four digit odd numbers that can be formed using 0, 1, 2, 3, 5, 7 (using repetition allowed) are:

(A) 216 (B) 375 (C) 400 (D) 720

Ans. :

d. 720

52. It is required to seat 5 men and 4 women in a row so that the women occupy the even places.The number of ways such arrangements are possible are

(A) 8820 (B) 2880 (C) 2088 (D) 2808

Ans. :

b. 2880

Solution:

Total number of persons are 9 in which there are 5 men and 4 women.

So total number of place = 9

Now women seat in even place.
So total number of arrangement = 4! (_W_W_W_W_) (W-Woman)
Men sit in odd place.
So total number of arrangement = 5! (MWMWMWMWM) (M-Man)
Now Total number of arrangement = 5! × 4! = 120 × 24 = 2880

53. The number of ways in which the letters of the word ARTICLE can be arranged so that even places are always occupied by consonants is:

(A) 576 (B) ${}^4C_3 \times 4!$ (C) $2 \times 4!$ (D) None of these.

Ans. :

a. 576

Solutions:

There are 3 even places in the 7 letter word ARTICLE. So, we have to arrange 4 consonants in these 3 places in ${}^4P_3$ ways. And the remaining 4 letters can be arranged among themselves in 4! ways.

$\therefore$ Total number of ways of arrangement = ${}^4P_3 \times 4! = 4! \times 4! = 576$

54. The number of rectangles that you can find on a chess board is:

(A) 1442 (B) 1296 (C) 1256 (D) None of these

Ans. :

b. 1296

55. The number of five-digit telephone numbers having at least one of their digits repeated is:

(A) 90000. (B) 100000. (C) 30240. (D) 69760

Ans. :

d. 69760

Solutions:

Total number of five digit numbers (since there is no restriction of the number 0XXXX) = 10 × 10 × 10 × 10 × 10 = 100000.

These numbers also include the numbers where the digits are not being repeated. So, we need to subtract all such numbers.

Number of 5 digit numbers that can be formed without any repetition of digits = 10 × 9 × 8 × 7 × 6 = 30240

$\therefore$ Number of five-digit telephone numbers having at least one of their digits repeated = {Total number of 5 digit numbers} - {Number of numbers that do not have any digit repeated} = 100000 - 30240 = 69760

56. There are 44 candidates for a Natural science scholarship, 22 for a Classical and 66 for a Mathematical scholarship, then find the no.of ways one of these scholarship can be awarded is:

(A) 6 (B) 10 (C) 48 (D) 12

Ans. :

d. 12

Solution:

Natural science scholarship can be awarded to anyone of the four candidates.So, there are 44 ways of awarding natural science scholarships.

Similarly, mathematical and classical scholarships can be awarded in 22 and 66 ways respectively.

By the fundamental principle of addition, number of ways of awarding one of the three scholarships is.

= 4 + 2 + 6 = 12

57. In a crossword puzzle, 20 words are to be guessed of which 88 words have each an alternative solution also.The number of possible solutions will be:

(A) ${}^{20}P_8$ (B) ${}^{20}C_8$ (C) 515 (D) 256

Ans. :

d. 256

Solution:

8 out of the 20 words have an alternative solution.

So 12 words are fixed, so they wont affect the number of ways.

So now we find the number of ways in which the remaining 8 words can be guessed.

Now every word have 2 solutions, so there are 2 ways to fill each of these words, and for 8 such words, it can be done in

$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times = 2^8$

$= 256$

58. In a chess tournament each of six players will play every other player exactly once. How many matches will be played during the tournament?

(A) 36 (B) 30 (C) 15 (D) 12

Ans. :

c. 15

Solution:

First player can play 5 matches with other five players.

Second player can play 4 matches with other four players and proceeding this way, the fifth player will playonly one match with sixth player.

$\therefore$ Total number of matches played = 5 +4 + 3 + 2 + 1 = 15.

Short cut : Number of matches played by n players $= \frac{n(n+1)}{2}$

59. How many 3 - letter words with or without meaning, can be formed out of the letters of the word, LOGARITHMS, if repetition of letters is not allowed:

(A) 720 (B) 420 (C) None of these (D) 5040

Ans. :

a. 720

Solution:

The word LOGARITHMS has 10 different letters.

Hence, the number of 3-letter words(with or without meaning) formed by using these letters.

$= {}^{10}\mathrm{P}_3$

$= 10 \times 9 \times 8$

$= 720$

60. The number of ways in which the letters of the word 'CONSTANT' can be arranged without changing the relative positions of the vowels and consonants is.

(A) 360 (B) 256 (C) 444 (D) None of these.

Ans. :

a. 360

Solutions:

The word CONSTANT consists of two vowels that are placed at the 2nd and 6th position, and six consonants.

The two vowels can be arranged at their respective places, i.e. 2nd and 6th place, in 2! ways.

The remaining 6 consonants can be arranged at their respective places in $\frac{6!}{2!\,2!}$ ways.

$\therefore$ Total number of arrangements $= 2! \times \frac{6!}{2!\,2!} = 360$

61. In a class there are 18 boys who are over 160 cm tall If these constitute three-fourths of the boys and the total number of boys is tow-third of the total number of students in the class what is the number of girls in the class?

(A) 6 (B) 12 (C) 18 (D) 24

Ans. :

c. 12

Solution:

Given in class there are 18 boys who over 160 cm tall and these are three-fourths of total boys

Then total boys in class $= 18 \div \frac{3}{4} = 18 \times \frac{4}{3} = 24$

And total no of student in class $= 24 \times \frac{3}{2} = 36$

Then number of girls $= 36 - 24 = 12$

62. A letter lock contains 5 rings each marked with for different letters.The number of all possible unsuccessful attempts to open the lock is:

(A) 625 (B) 1024 (C) 624 (D) 1023 b

Ans. :

b. 1024

63. The number of six letter words that can be formed using the letters of the word "ASSIST" in which S's alternate with other letters is:

(A) 12 (B) 24 (C) 18 (D) None of these.

Ans. :

a. 12

Solutions:

All S's can be placed either at even places or at odd places, i.e. in 2 ways. The remaining letters can be placed at the remaining places in 3!, i.e. in 6 ways.

$\therefore$ Total number of ways = 6 × 2 × 2 = 12

64. A person wishes to make up as many different parties as he can out of 20 friends. Each party consists of the same number of friends. How many should be invited at a time:

(A) 8 (B) 9 (C) 10 (D) 11

Ans. :

c. 10

65. Given 11 points, of which 5 lie on one circle, other than these 5, no 4 lie on one circle. Then the number of circles that can be drawn so that each contains at least 3 of the given points is:

(A) 216 (B) 156 (C) 172 (D) None of these.

Ans. :

b. 156

Solution:

We need at least three points to draw a circle that passes through them.

Now, number of circles formed out of 11 points by taking three points at a time $= {}^{11}\mathrm{C}_3 = 165$

Number of circles formed out of 5 points by taking three points at a time $= {}^{5}\mathrm{C}_3 = 10$

It is given that 5 points lie on one circle.

Required number of circles = 165 - 10 + 1 = 156.

66. Total number of words formed by 2 vowels and 3 consonants taken from 4 vowels and 5 consonants is equal to:

(A) 60 (B) 120 (C) 7200 (D) None of these.

Ans. :

c. 7200

Solution:

2 out of 4 vowels can be chosen in ${}^4C_2$ ways and 3 out of 5 consonants can be chosen in ${}^5C_3$ ways.

Thus, there are $({}^4\mathrm{C}_2 \times {}^5\mathrm{C}_3)$ groups, each containing 2 vowels and 3 consonants.

Each group contains 5 letters that can be arranged in 5! ways.

Required number of words $= ({}^4\mathrm{C}_2 \times {}^5\mathrm{C}_3) \times 5!$

$= 60 \times 120 = 7200$

67. In how many ways 8 distinct toys can be distributed among 5 children?

(A) ${}^8\mathrm{P}_5$ (B) ${}^5\mathrm{P}_8$ (C) 5^8 (D) 8^5

Ans. :

c. 5^8

Solution:

Given that, the number of toys $= 8$

The number of children $= 5$.

Hence, the number of ways 8 distinct toys can be distributed among 5 children is $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^8$.

Hence, option (c) 5^8 is the correct answer.

68. On the occasion of Deepawali festival, each student of a class sends greeting cards to the others.If there are 20 students in the class, then the total number of greeting cards exchanged by the students is:

(A) ${}^{20}\mathrm{C}_2$ (B) $2.\,{}^{20}\mathrm{C}_2$ (C) $2.\,{}^{20}\mathrm{P}_2$ (D) None of these

Ans. :

b. $2.\,{}^{20}\mathrm{C}_2$

69. A committee of 7 has to be formed from 9 boys and 4 girls. In how many ways can this be done when the committee consists of exactly 3 girls:

(A) 540 (B) 405 (C) 504 (D) None of these

Ans. :

c. 504

Solution:

Given number of boys = 9

Number of girls = 4

Now, A committee of 7 has to be formed from 9 boys and 4 girls.

Now, If in committee consist of exactly 3 girls:

${}^4\mathrm{C}_3 \times {}^9\mathrm{C}_4$

$= \left(\frac{4!}{(3! \times 1!)}\right) \times \left(\frac{9!}{(4! \times 5!)}\right)$

$$= \left(\frac{4 \times 3}{(3!)}\right) \times \left(\frac{9 \times 8 \times 7 \times 6 \times 5!}{(4! \times 5!)}\right)$$
$$= 4 \times \frac{(9 \times 8 \times 7 \times 6)}{4!}$$
$$= 4 \times \frac{(9 \times 8 \times 7 \times 6)}{(4 \times 3 \times 2 \times 1)}$$
$$= 9 \times 8 \times 7$$
$$= 504$$

70. Choose the correct answer.
The number of parallelograms that can be formed from a set of four parallel lines intersecting another set of three parallel lines is.

(A) 6 (B) 18 (C) 12 (D) 9

Ans. :

b. 18

Solution:

To form parallelogram we required a pair of line from a set of 4 lines and another pair of line from another set of 3 lines.

Required number of parallelograms = ${}^4C_2 \times {}^3C_2 = 6 \times 3 = 18$

71. If a represents the number of permutations of (x + 2) things taken together b represents the number of permutation of 11 things taken together out of x things, and c represents the number of permutation of(x – 11)things taken together so that a = 182, bc = then x is equal to:

(A) 15 (B) 12 (C) 10 (D) 18

Ans. :

b. 12

72. If ${}^{n}C_{15} = {}^{n}C_{6}$ then the value of ${}^{n}C_{21}$ is:

(A) 0 (B) 1 (C) 21 (D) None of these

Ans. :

b. 0

Solution:

We know that

if ${}^{n}Cr_1 = {}^{n}Cr_2$

$\Rightarrow n = r^1 + r^2$

Given, ${}^{n}C_{15} = {}^{n}C_{6}$

$\Rightarrow n = 15 + 6$

$\Rightarrow n = 21$

Now, ${}^{21}C_{21} = 1$

73. If ${}^{20}C_{3r+1} = {}^{20}C_{r\text{-}1}$, is then r equal to:

(A) 10 (B) 11 (C) 19 (D) 12

Ans. :

a. 10

Solution:

$r+1+r-1=20 \;\; [\because \; {}^{n}C_{x} = {}^{n}C_{y} \Rightarrow n = x+y \text{ or } x=y]$

$\Rightarrow 2r = 20$

$\Rightarrow r = 10$

74. If $C_0 + C_1 + C_2 + \ldots + C_n = 256$, then ${}^{2n}C_2$ is equal to:

(A) 56 (B) 120 (C) 28 (D) 91

Ans. :

b. 120

Solution:

If set S has n elements, then C (n, k)C n, k is the number of ways of choosing k elements from S.

Thus, the number of subsets of SS of all possible values is given by,

$C(n,0)+C(n,1)+C(n,3)+\ldots\ldots+C(n,n)=2^{n}$

Comparing the given equation with the above equation:

$2^{n}=256$

$\Rightarrow 2^{n}=2^{8}$

$\Rightarrow n=8$

$\therefore {}^{2n}C_{2}={}^{16}C_{2}$

$\Rightarrow {}^{16}C_{2}=\frac{16!}{2!4!}=\frac{16\times15}{2}=120$

75. How many numbers greater than 10 lacs be formed from 2, 3, 0, 3, 4, 2, 3?

(A) 420 (B) 360 (C) 400 (D) 300

Ans. :

b. 360

Solutions:

10 lakhs consists of seven digits.

Number of arrangements of seven numbers of which 2 are similar of first kind, 3 are similar of second kind $=\frac{7!}{2!\,3!}$

But, these numbers also include the numbers in which the first digit has been considered as 0. This will result in a number less than 10 lakhs. Thus, we need to subtract all those numbers.

Numbers in which the first digit is fixed as 0 = Number of arrangements of the remaining 6 digits $=\frac{6!}{2!\,3!}$

Total numbers greater than 10 lakhs that can be formed using the given digits $=\frac{7!}{2!\,3!}-\frac{6!}{2!\,3!}$

$=420-60$

$=360$

76. The number of arrangements of the letters of the word BHARAT taking 3 at a time is:

(A) 72 (B) 120 (C) 14 (D) None of these.

Ans. :

a. 72

Solutions:

When we make words after selecting letters of the word BHARAT, it could consist of a single A, two As or no A.

Case-I: A is not selected for the three letter word.

Number of arrangements of three letters out of B, H, R and T = 4 × 3 × 24 × 3 × 2 = 24

Case-II: One A is selected and the other two letters are selected out of B, H, R or T. Possible ways of selection: Selecting two letters out of B, H, R or T can be done in ${}^4P_2$ = 12 ways. Now, in each of these 12 ways, these two letters can be placed at any of the three places in the three letter word in 3 ways.

∴ Total number of words that can be formed = 12 × 3 = 36

Case-III: Two A's and a letter from B, H, R or T are selected.

Possible ways of arrangement:

Number of ways of selecting a letter from B, H, R or T = 4 And now this letter can be placed in any one of the three places in the three letter word other than the two A's in 3 ways.

∴ Total number of words having 2 A's = 4 × 3 = 12

Hence, total number of words that can be formed = 24 + 36 + 12 = 72

77. There were two women participants in a chess tournament.The number of games the men played between themselves exceeded by 52 the number of games they played with women.If each player played one game with each other, the number of men in the tournament, was:

(A) 10 (B) 11 (C) 12 (D) 13

Ans. :

d. 13

78. How many numbers of 4 - digits can be formed by using the digits 1, 2, 3, 4, 5, 6, 7 if atleast one digit is repeated:

(A) ${}^7P_4$ (B) 7^4 (C) $7^4 - {}^7p_4$ (D) None of these

Ans. :

c. $7^4 - {}^7p_4$

79. There are 12 points in a plane. The number of the straight lines joining any two of them when 3 of them are collinear is:

(A) 62 (B) 63 (C) 64 (D) 65

Ans. :

c. 64

Solution:

Number of straight lines joining 12 points if we take 2 points at a time $= {}^{12}C_2$

$= \frac{12!}{2!10!} = 66$

Number of straight lines joining 3 points if we take 2 points at a time $= {}^{3}C_2 = 3$

Required number of straight lines = 66 - 3 + 1 = 64

80. A bag contains 3 black, 4 white and 2 red balls, all the ballsbeing different.Number of selections of atmost 6 balls containing balls of all the colours is:

(A) 1008 (B) 1080 (C) 1204 (D) 1130

Ans. :

a. 1008

81. Match the terms given in Column-I with the terms given in Column-II and choose the correct option from the codes given below.

	Column-I		Column-II
(A)	If $P(n,4) = 20.P(n,2)$ then the value of n is	(1)	28
(B)	${}^{5}p_r = {}^{26}p_{r\text{-}1}$	(2)	4
(C)	${}^{5}p_r = {}^{6}p_{r\text{-}1}$	(3)	7
(D)	Value of $\frac{8!}{6! \times 2!}$ is	(4)	3

Codes

ABCD

(A) 4321 (B) 3412 (C) 4231 (D) 3421

Ans. :

d. 3421

82. If ${}^{n}C_9 = {}^{n}C_8$, what is the value of ${}^{n}C_{17}$

(A) 1 (B) 0 (C) 3 (D) 17

Ans. :

a. 1

83. If ${}^{n+1}C_3 = 2.{}^{n}C_2$, then n:

(A) 3 (B) 4 (C) 5 (D) 6

Ans. :

c. 5

Solution:

${}^{n+1}C_3 = 2 \times {}^{n}C_2$

$\Rightarrow \frac{(n+1)!}{3!(n-2)!} = 2 \times \frac{n!}{2!(n-1)!}$

$\Rightarrow n+1 = 6$

$\Rightarrow n = 5$

84. The number of ways in which four letters of the word MATHEMATICS can be arranged is given by:

(A) 136 (B) 192 (C) 1680 (D) 2454

Ans. :

d. 2454

85. Arrange the given words in the sequence in which they occur in the dictionary and then choose the correct sequence.

1.Page 2.Pagan 3.Palisade 4.Pageant 5.Palate

(A) 1, 4, 2, 3, 5 (B) 2, 4, 1, 3, 5 (C) 2, 1, 4, 5, 3 (D) 1, 4, 2, 5, 3

Ans. :

c. 2, 1, 4, 5, 3

Solution:

Words 1, 2 and 4 have first 3 letter as pag and words 3 and 5 have first 3 letters as pal.

So words starting with pal will come later than words starting with pag as in Alphabet set the letter g comes after letter a.

The fourth letter of word 5 is a and of word 3 is i hence word 3 will be the last in order.

So choice A and B are incorrect and the answer will be either C or D.

Simply arranging the 4th letter of words 1, 2 and 4 will give the correct sequence choice as C.

86. The total number of 9 - digit numbers which have all different digits is:

(A) 10! (B) 9! (C) 99x! (D) 10x, 10!

Ans. :

c. 99x!

87. At the end of a business conference, the ten people present all shake hands with each other once. How many handshakes will there be altogether?

(A) 20 (B) 45 (C) 55 (D) 90

Ans. :

b. 45

Solution:

Each person shakes hands with others. Calculation: Total number of handshakes = ${}^{10}C_2$ = $\frac{(10\times9)}{2}$ = 45 handshakes.

88. There are 10 true - false questions in an examination.These questions can be answered in:

(A) 20 ways. (B) 100 ways. (C) 512 ways. (D) 1024 ways.

Ans. :

d. 1024 ways.

Solution:

Given that there are 10 questions.

Each question can be answered in two ways. (i.e. either true or false).

Hence, the number of ways these questions can be answered is 2^{10}, which is equal to 1024.

89. There are 15 points in a plane, no two of which are in a straight line except 4, all of which are in a straight line.The number of triangle that can be formed by using these 15 points is:

(A) ${}^{15}\mathrm{C}_3$ (B) 490 (C) 451 (D) 415

Ans. :

c. 451

Solution:

The required number of triangle ${}^{15}\mathrm{C}_3 - {}^{4}\mathrm{C}_3 = 455 - 4 = 451$

90. There are ′m′ copies each of ′n′ different books in a university library. The number of ways in which one or more than one book can be selected is:

(A) $m^n - 1$ (B) $(m + 1)^n - 1$ (C) $(m + 1)^n - m^n$ (D) $(m + 1)^n - m$

Ans. :

b. $(m + 1)^n - 1$

91. Choose the correct answer.

A five digit number divisible by 3 is to be formed using the numbers 0, 1, 2, 3, 4 and 5 without repetitions. The total number of ways this can be done is.

(A) 216 (B) 600 (C) 240 (D) 3125

Ans. :

a. 216

Solution:

We know that a number is divisible by 3 if the sum of its digits is divisible by 3.

Now sum of the given six digits is 15 which is divisible by 3. So to form a number of five-digit which is divisible by 3 we can remove either ‘O’ or ‘3’. If digits 1, 2, 3, 4, 5 are used then number of required numbers = 5!

If digits 0, 1, 2, 4, 5 are used then first place from left can be filled in 4 ways and remaining 4 places can be filled in 4! ways. So in this case required numbers are 4 × 4! ways.

So, total number of numbers = 120 + 96 = 216

92. Choose the correct answer.
Given 5 different green dyes, four different blue dyes and three different red dyes, the number of combinations of dyes which can be chosen taking at least one green and one blue dye is.

(A) 3600 (B) 3720 (C) 3800 (D) 3600

Ans. :

b. 3720

Solution:

At least one green dye can be chosen in $^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5 = 22^5 - 1$ ways.

At least one blue dye can be chosen in $^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4 = 2^4 - 1$ ways.

Any number of red dyes can be chosen in $^3C_0 + {}^3C_1 + {}^3C_2 + {}^3C_3 = 2^3$ ways.

so, total nnumber of required selection $= (2^5 - 1) \times (2^4 - 1) \times 2^3 = 3720$

93. The letters of the word RACHIT are written in all possible manner and words are written as in dictionary. The rank of word RACHIT is:

(A) 365 (B) 702 (C) 481 (D) 480

Ans. :

c. 481

94. $^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5$ is equal to:

(A) 30 (B) 31 (C) 32 (D) 33

Ans. :

b. 31

Solution:

$^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5$

$= {}^5C_1 + {}^5C_2 + {}^5C_2 + {}^5C_1 + {}^5C_5$

$= 2 \times {}^5C_1 + 2 \times {}^5C_2 + {}^5C_5$

$= 2 \times 5 + 2 \times \frac{5!}{2!3!} + 1$

$= 10 + 20 + 1$

$= 31$

95. There are 13 players of cricket, out of which 4 are bowlers. In how many ways a team of eleven be selected from them so as to include at least two bowlers?

(A) 72 (B) 78 (C) 42 (D) None of these.

Ans. :

b. 78

Solution:

4 out of 13 players are bowlers.

In other words, 9 players are not bowlers.

A team of 11 is to be selected so as to include at least 2 bowlers.

Number of ways $= {}^4\mathrm{C}_2 \times {}^9\mathrm{C}_9 + {}^4\mathrm{C}_3 \times {}^9\mathrm{C}_8 + {}^4\mathrm{C}_4 \times {}^9\mathrm{C}_7$

$= 6 + 36 + 36$

$= 78$

96. A lady gives a dinner party for six guests. The number of ways in which they may be selected from among ten friends if two of the friends will not attend the party together is:

(A) 112 (B) 140 (C) 164 (D) None of these.

Ans. :

b. 140

Solution:

Suppose there are two friends, A and B, who do not attend the party together.

If both of them do not attend the party, then the number of ways of selecting 6 guests $= {}^8\mathrm{C}_6 = 28$

If one of them attends the party, then the number of ways of selecting 6 guests $= 2.{}^8\mathrm{C}_5 = 112$

Total number of ways = 112 + 28 = 140

97. If $= {}^{43}\mathrm{C}_{\mathrm{r}\text{-}6} = {}^{43}\mathrm{C}_{3\mathrm{r}+1}$, then the value of r is is:

(A) 12 (B) 8 (C) 6 (D) 10

Ans. :

a. 12

Solution:

$\mathrm{r} - 6 + 3\mathrm{r} + 1 = 43$

$\Rightarrow 4\mathrm{r} - 5 = 43$

$\Rightarrow 4\mathrm{r} = 48$

$\Rightarrow \mathrm{r} = 12$

98. A dictionary is printed consisting of 7 lettered words only that can be made with a letter of the word $CRICKET$. If the words are printed at the alphabetical order, as in an ordinary dictionary, then the number of word before the word $CRICKET$ is

(A) 530 (B) 480 (C) 531 (D) 481

Ans. : a

(a) The number of words before the word $CRICKET$ is $4 \times 5! + 2 \times 4! + 2! = 530$.

99. The number of positive integral solutions of $abc = 30$ is

(A) 30 (B) 27 (C) 8 (D) None of these

Ans. : b

(b) We have, $30=2\times3\times5$. So, 2 can be assigned to either a or b or c $i.e. 2$ can be assigned in 3 ways.

Similarly, each of 3 and 5 can be assigned in 3 ways.

Thus, the no. of solutions are $3\times3\times3=27$.

100. 12 persons are to be arranged to a round table. If two particular persons among them are not to be side by side, the total number of arrangements is

(A) $9(10\ !)$ (B) $2(10\ !)$ (C) $45(8\ !)$ (D) $10\ !$

Ans. : a

(a) 12 persons can be seated around a round table in $11\ !$ ways.

The total number of ways in which 2 particular persons sit side by side is $10\ !\ \times\ 2\ !$.

Hence the required number of arrangements $=11\ !\ -10\ !\ \times2\ !\ =9\times10\ !$.

101. There are $(n+1)$ white and $(n+1)$ black balls each set numbered 1 to $n+1$. The number of ways in which the balls can be arranged in a row so that the adjacent balls are of different colours is

(A) $(2n+2)\ !$ (B) $(2n+2)\ !\ \times2$ (C) $(n+1)\ !\ \times2$ (D) $2\{(n+1)\ !\}^2$

Ans. : d

(d) Since the balls are to be arranged in a row so that the adjacent balls are of different colours, therefore we can begin with a white ball or a black ball.

If we begin with a white ball, we find that $(n+1)$ white balls numbered 1 to $(n+1)$ can be arranged in a row in $(n+1)\ !$ ways.

Now $(n+2)$ places are created between $n+1$ white balls which can be filled by $(n+1)$ black balls in $(n+1)\ !$ ways.

So the total number of arrangements in which adjacent balls are of different colours and first ball is a white ball is $(n+1)\ !\ \times(n+1)\ !\ =[(n+1)\ !]^2$.

But we can begin with a black ball also.

Hence the required number of arrangements is $2[(n+1)\ !]^2$.

102. How many words can be made out from the letters of the word $INDEPENDENCE$, in which vowels always come together

(A) 16800 (B) 16630 (C) 1663200 (D) None of these

Ans. : a

(a) Required number of ways are $\frac{8\ !}{2\ !\ 3\ !}\times\frac{5\ !}{4\ !}=16800$.

{Since $IEEEENDPNDNC=8$ letters}.

103. The number of ways in which the letters of the word $ARRANGE$ can be arranged such that both R do not come together is

(A) 360 (B) 900 (C) 1260 (D) 1620

Ans. : b

(b) The word $ARRANGE$, has AA, RR, NGE letters, that is two A' s, two R's and

N, G, E one each.

$\therefore$ The total number of arrangements

$=\frac{7!}{2!2!1!1!1!}=1260$

But, the number of arrangements in which both RR are together as one unit = $\frac{6!}{2!1!1!1!1!}=360$

$\therefore$ The number of arrangements in which both RR do not come together $=1260-360=900$.

104. If ${}^{56}P_{r+6}:{}^{54}P_{r+3}=30800:1$, then $r=$

(A) 31 (B) 41 (C) 51 (D) None of these

Ans. : b

(b) $\frac{56!}{(50-r)!}\times\frac{(51-r)!}{54!}$

$=\frac{30800}{1}\Rightarrow 56\times 55\times(51-r)=30800$

$\Rightarrow r=41$.

105. The number of words which can be made out of the letters of the word $MOBILE$when consonants always occupy odd places is

(A) 20 (B) 36 (C) 30 (D) 720

Ans. : b

(b) The word $MOBILE$ has three even places and three odd places. It has 3 consonants and 3 vowels. In three odd places we have to fix up 3 consonants which can be done in ${}^{3}P_{3}$ ways.

Now, remaining three places we have to fix up remaining three places we have to fix up remaining three which can be done in ${}^{3}P_{3}$ ways.

The total number of ways $={}^{3}P_{3}\times{}^{3}P_{3}=36$.

106. The number of ways in which the letters of the word $TRIANGLE$ can be arranged such that two vowels do not occur together is

(A) 1200 (B) 2400 (C) 14400 (D) None of these

Ans. : c

(c) $\bullet T\bullet R\bullet N\bullet G\bullet L$

Three vowels can be arrange at 6 places in ${}^{6}P_{3}=120$ ways. Hence the required number of arrangements $=120\times 5!=14400$.

107. The letters of the word $MODESTY$ are written in all possible orders and these words are written out as in a dictionary, then the rank of the word $MODESTY$ is

(A) 5040 (B) 720 (C) 1681 (D) 2520

Ans. : c

(c) Words start with D are $6!=720$,

start with E are 720, start with MD are $5!=120$ and start with ME are 120.

Now the first word starts with MO is nothing but $MODESTY$.

Hence rank of $= 720 + 720 + 120 + 120 + 1 = 1681$.

108. Four dice (six faced) are rolled. The number of possible outcomes in which at least one die shows 2 is

(A) 1296 (B) 625 (C) 671 (D) None of these

Ans. : c

(c) The number of possible outcomes with 2 on at least one die = (The total number of outcomes) -(The number of outcomes in which 2 does not appear on any die) $6^4 - 5^4 = 1296 - 625 = 671$.

109. The number of ways in which 9 persons can be divided into three equal groups is

(A) 1680 (B) 840 (C) 560 (D) 280

Ans. : a

(a) Total ways $= \frac{9!}{(3!)^3} = \frac{9\times8\times7\times6\times5\times4}{3\times2\times3\times2} = 1680$.

110. We are to form different words with the letters of the word $INTEGER$. Let m_1 be the number of words in which I and N are never together and m_2 be the number of words which begin with I and end with R, then m_1/m_2 is equal to

(A) 30 (B) 60 (C) 90 (D) 180

Ans. : a

(a) We have 5 letters other than 'I' and 'N' of which two are identical $(E's)$.

We can arrange these letters in a line in $\frac{5!}{2!}$ ways.

In any such arrangement, 'I' and 'N' can be placed in 6 available gaps in ${}^6P_2$ ways,

so required number $= \frac{5!}{2!} \cdot {}^6P_2 = m_1$.

Now, if word start with 'I' and end with 'R' then the remaining letters are 5.

So total no. of ways $= \frac{5\,!}{2\,!} = m_2$

$\therefore \frac{m_1}{m_2} = \frac{5!}{2!} \cdot \frac{6!}{4!} \cdot \frac{2!}{5!} = 30$.

111. If the letters of the word $KRISNA$ are arranged in all possible ways and these words are written out as in a dictionary, then the rank of the word $KRISNA$ is

(A) 324 (B) 341 (C) 359 (D) None of these

Ans. : a

(a) Words starting from A are 5 != 120

Words starting from I are $5! = 120$

Words starting from KA are $4! = 24$

Words starting from KI are $4! = 24$

Words starting from KN are $4! = 24$

Words starting from KRA are $3! = 6$

Words starting from $KRIA$ are $2! = 2$
Words starting from $KRIN$ are $2! = 2$
Words starting from $KRISA$ are $1! = 1$
Words starting from $KRISNA$ are $1! = 1$
Hence rank of the word $KRISNA$ is 324.

112. How many words can be made from the letters of the word $INSURANCE$, if all vowels come together

(A) 18270 (B) 17280 (C) 12780 (D) None of these

Ans. : d

(d) $IUAENSRNC$
Obviously required number of words are
$\frac{6\,!}{2\,!} \times 4\,! \ = 8640\,.$

*** Given section consists of questions of 2 marks each.** **[40]**

113. How many 3-digit even numbers can be formed, from the digits 1, 2, 3, 4, 5, 6 if the digits can be repeated?

Ans. : Here the unit place can be filled by any one of the digits 2, 4, 6. So the unit place can be filled in 3 ways. Now the tens and hundreds place can be filled by any one of the digits 1, 2, 3, 4, 5, 6. So the tens and hundreds place can be filled in 6 ways each.
$\therefore$ Total number of 3-digit even numbers $= 6 \times 6 \times 3 = 108$

114. Find the number of 4-digit numbers that can be formed using the digits 1, 2, 3, 4, 5 if no digit is repeated. How many of these will be even?

Ans. : Here total number of digits = 5
Number of digits used (no digit is repeated) = 4
$\therefore$ Number of permutations = $^5P_4$
$= \frac{5!}{1!}$ = 5 $\times$ 4 $\times$ 3 $\times$ 2 $\times$ 1 = 120
Now the unit's place can be filled with any one of the digits 2, 4 for even number.
$\therefore$ Number of permutations = $^2P_1$ = 2
Now the remaining three places can be filled with the remaining 4 digits.
$\therefore$ Number of permutation = $^4P_3$
$\frac{4!}{1!}$ = 4 $\times$ 3 $\times$ 2 $\times$ 1 = 24
Hence total number of permutations of even numbers = 2 $\times$ 24 = 48

115. From a committee of 8 persons in how many ways can we choose a chairman and a vice chairman assuming one person cannot hold more than one position?

Ans. : Here total number of persons = 8
Number of persons used (no persons is repeated) = 2

$\therefore$ Number of permutation $= {}^8P_2$

$= \frac{8!}{6!} = \frac{8\times7\times6!}{6!} = 56$

116. Determine n if ${}^{2n}C_3 : {}^nC_2 = 12:1$

Ans. : Here,we have ${}^{2n}C_3 : {}^nC_3 = 12:1$

$\Rightarrow \frac{2n_{C_3}}{n_{C_3}} = \frac{12}{1}$

$\Rightarrow \frac{\frac{2n!}{3!(2n-3)!}}{\frac{n!}{3!(n-3)!}} = \frac{12}{1}$

$\Rightarrow \frac{\frac{2n\times(2n-1)\times(2n-2)\times(2n-3)!}{3!(2n-3)!}}{\frac{n\times(n-1)\times(n-2)\times(n-3)!}{3!(n-3)!}} = \frac{12}{1}$

$\Rightarrow \frac{\frac{2n\times(2n-1)\times(2n-2)}{3!}}{\frac{n\times(n-1)\times(n-2)}{3!}} = \frac{12}{1}$

$\Rightarrow \frac{2n\times(2n-1)\times(2n-2)}{n\times(n-1)\times(n-2)} = \frac{12}{1}$

$\Rightarrow \frac{2n\times(2n-1)\times2\times(n-1)}{n\times(n-1)\times(n-2)} = \frac{12}{1}$

$\Rightarrow \frac{4\times n\times(2n-1)}{n\times(n-2)} = \frac{12}{1}$

$\Rightarrow \frac{4\times(2n-1)}{(n-2)} = \frac{12}{1}$

$\Rightarrow 4\times(2n-1) = 12\times(n-2)$

$\Rightarrow$ 8 n - 4 = 12 n - 24

$\Rightarrow$ 12 n - 8 n = 24 - 4

$\Rightarrow$ 4 n = 20

$\therefore$ n = 5

117. Determine n if ${}^{2n}C_3 \ : \ {}^nC_3 = 11:1$

Ans. : Here,we have ${}^{2n}C_3 \ : \ {}^nC_3 = 11:1$

$\Rightarrow \frac{2n_{C_3}}{n_{C_3}} = \frac{11}{1}$

$\Rightarrow \frac{(2n)!}{3!(2n-3)!} \times \frac{3!(n-3)!}{n!} = \frac{11}{1}$

$\Rightarrow \frac{(2n)(2n-1)(2n-2)(2n-3)!}{3!(2n-3)!} \times \frac{3!(n-3)!}{n(n-1)(n-2)(n-3)!} = \frac{11}{1}$

$\Rightarrow \frac{\frac{2n\times(2n-1)\times(2n-2)}{3!}}{\frac{n\times(n-1)\times(n-2)}{3!}} = \frac{11}{1}$

$\Rightarrow \frac{(2n)(2n-1)(2n-2)}{n(n-1)(n-2)} = \frac{11}{1}$

$\Rightarrow \frac{2n\times(2n-1)\times2\times(n-1)}{n\times(n-1)\times(n-2)} = \frac{11}{1}$

$\Rightarrow \frac{4\times n\times(2n-1)}{n\times(n-2)} = \frac{11}{1}$

$\Rightarrow \frac{4(2n-1)}{n-2} = \frac{11}{1}$

$\Rightarrow 4\times(2n-1) = 11\times(n-2)$

$\Rightarrow 8n-4=11n-22$
$\Rightarrow$ 3n = 18
$\therefore$ n = 6

118. In how many ways can a student choose a programme of 5 courses if 9 courses are available and 2 specific courses are compulsory for every student?

Ans. : There are 9 courses and number of courses to be selected are 5 in which 2 specific courses are compulsory.
We have to select 3 courses out of remaining 7 courses.
$\therefore$ Number of ways of selection $={}^7C_3$
$\frac{7!}{3!4!}=\frac{7\times6\times5\times4!}{3\times2\times1\times4!}=35$

119. In how many ways can the letters of the word ASSASSINATION be arranged so that all the S's are together?

Ans. : Here total letters are 13 in the word ASSASSINATION in which A appears 3 times, S appears 4 times, 1 appears 2 times and N appears 2 times. Now four S's taken together become a single letter and other remaining letters taken with this single letter.
$\therefore$ Number of arrangements $=\frac{10!}{3!2!2!}=\frac{10\times9\times8\times7\times6\times5\times4\times3!}{3!2\times1\times2\times1}$
$\Rightarrow 10\times9\times8\times7\times6\times5=151200$

120. How many 2 digit even numbers can be formed from the digits 1, 2, 3, 4, 5 if the digits can be repeated?

Ans. : There will be as many ways as there are ways of filling 2 vacant places, blank in succession by the five given digits. Here, it is given that in this case, we start filling in unit’s place, because the options for this place are 2 and 4 only and this can be done in 2 ways; following which the ten’s place can be filled by any of the 5 digits in 5 different ways as the digits can be repeated. Therefore, by the multiplication principle, therefore, the required number of two digits even numbers is 2 $\times$ 5, i.e., 10.

121. Find the number of different signals that can be generated by arranging at least 2 flags in order (one below the other) on a vertical staff, if five different flags are available.

Ans. : Here, we have to find the number of different signals that can be generated by arranging at least 2 flags in order i.e., a signal can consist of either 2 flags, 3 flags, 4 flags or 5 flags. If a signal consists of 2 flags, then vacant places are 2 and 5 flags are available.
$\therefore$ Number of ways of filling first vacant place = 5
Number of ways of filling second vacant place = 4
Then, the total number of signals consisting by 2 flags
= 5 $\times$ 4 = 20
Now, if a signal consists of 3 flags, then vacant places are 3 and 5 flags are

available.

$\therefore$ Number of ways of filling of first vacant place = 5

Number of ways of filling of second vacant place = 4

Number of ways of filling of third vacant place = 3

Then, the total number of signals consisting by 3 flags

= $5 \times 4 \times 3$ = 60

Similarly, the total number of signals consisting of 4 flags

= $5 \times 4 \times 3 \times 2$ = 120

Total number of signals consisting of 5 flags

= $5 \times 4 \times 3 \times 2 \times 1$ = 120

Now, different signals can be generated by arranging either 2 flags or 3 flags or 4 flags or 5 flags. So, by the fundamental principle of addition, we get

Total number of signals = 20 + 60 + 120 + 120 = 320

122. If $\frac{1}{8!}+\frac{1}{9!}=\frac{x}{10!}$, find x.

Ans. : Given, $\frac{1}{8!}+\frac{1}{9\times 8!}=\frac{x}{10\times 9\times 8!}$

Therefore 1 + $\frac{1}{9}=\frac{x}{10\times 9}$ or $\frac{10}{9}=\frac{x}{10\times 9}$

Thus, x = 100

123. How many numbers lying between 100 and 1000 can be formed with the digits 0, 1, 2, 3, 4, 5, if the repetition of the digits is not allowed?

Ans. : We know that,every number between 100 and 1000 is a 3-digit number. We, first, have to count the permutations of 6 digits taken 3 at a time. This number would be $^6P_3$. But, these permutations will include those also where 0 is at the 100's place. For example, 092, 042, . . ., etc are such numbers which are actually 2-digit numbers and therefore the number of such numbers has to be subtracted from $^6P_3$ to get the required number. To get the number of such numbers, we fix 0 at the 100's place and rearrange the remaining 5 digits taking 2 at a time. This number is $^5P_2$. Therefore,

The required number will be $={}^6P_3-{}^5P_2=\frac{6!}{3!}-\frac{5!}{3!}$

= $4 \times 5 \times 6 - 4 \times 5$ = 100

124. Find the number of different 8-letter arrangement that can be made from the letters of the word DAUGHTER so that all vowels occur together.

Ans. : We know that There are 8 different letters in the word DAUGHTER, in which there are 3 vowels, namely, A, U and E.

Since the vowels have to occur together, we can for the time being, assume them as a single object (AUE).

This single object together with 5 remaining letters (objects) will be counted as 6 objects.

Then we count permutations of these 6 objects taken all at a time.

Therefore, this number would be $^6P_6$ = 6!. Corresponding to each of these permutations, we shall have 3! permutations of the three vowels A, U, E taken all at a time.

Therefore, by the multiplication principle, the required number of permutations is = 6 ! $\times$ 3! = 4320

125. In how many ways can 4 red, 3 yellow and 2 green discs be arranged in a row if the discs of the same colour are indistinguishable?

Ans. : We know that the total number of discs are 4 + 3 + 2 = 9. Out of 9 discs, 4 are of the first kind (red), 3 are of the second kind (yellow) and 2 are of the third kind (green).

Thus, the number of arrangements $\frac{9!}{4!\,3!\,2!} = 1260$

126. If $^nC_9 = {^nC_8}$, find $^nC_{17}$

Ans. : We have, $^nC_9 = {^nC_8}$

i.e., $\frac{n!}{9!(n-9)!} = \frac{n!}{(n-8)!8!}$

or $\frac{1}{9} = \frac{1}{n-8}$ or n - 8 = 9 or n = 17

Thus, $^nC_{17} = {^{17}C_{17}} = 1$

127. A committee of 3 persons is to be constituted from a group of 2 men and 3 women. In how many ways can this be done? How many of these committees would consist of 1 man and 2 women?

Ans. : Here, order does not matter. Thus, we need to count combinations. There will be as many committees as there are combinations of 5 different persons taken 3 at a time. Therefore, the required number of ways = $^5C_3 = \frac{5!}{3!2!} = \frac{4\times5}{2} = 10$.

Now, 1 man can be selected from 2 men in $^2C_1$ ways and 2 women can be selected from 3 women in $^3C_2$ ways. Thus, the required number of committees = $^2\mathrm{C}_1 \times {^3\mathrm{C}_2} = \frac{2!}{1!!} \times \frac{3!}{2!1!} = 6$

128. From among the 36 teachers in a school, one principal and one vice-principal are to be appointed. In how many ways can this be done?

Ans. : The total number of teachers in a school = 36

One principal and one uice-principal are to be appointed.

$\therefore$ Total of ways = Number of arrangement of 36 things taken two at a time $= {^{36}\mathrm{P}_2}$

$= \frac{36!}{(36-2)!}$

$= \frac{36!}{34!}$

$= \frac{36\times35\times34!}{34!}$

$= 36 \times 35$

$= 1260$

Hence, Total number of ways to appoint one principal and one vice-principal are 1260.

129. In a class there are 27 boys and 14 girls. The teacher wants to select 1 boy and 1 girl to represent the class in a function. In how many ways can the teacher make this selection?

Ans. : Here the teacher is to perform two jobs.

i. Selecting a boy among 27 boys.

ii. Selecting a girl among 14 boys.

The first of these can be perfomed in 27 ways and the second in 14 ways.

Therefore by the fumdam pricipal of multiplication, the required number of ways is 27 × 14 = 378

Hence, the teacher can make the selection of a boy a girl in 378 ways.

130. There are four parcels and five post-offices. In how many different ways can the parcels be sent by registered post?

Ans. : Total number of parcels = 4

Total number of post-offices = 5

Since a percel can be sent to any one of the five post offices.

So, the requied number of ways = 5 × 5 × 5 × 5

= 5^4

= 625

Hence, total number of ways is 625.

131. Compute:

$\frac{11!-10!}{9!}$

Ans. : We have,

$\frac{11!-10!}{9!} = \frac{11\times10\times9!-10\times9!}{9!}$

$= \frac{9!\times10[11-1]}{9!}$

$= 10 \times 10$

$= 100$

Hence, $\frac{11!-10!}{9!} = 100$

132. Evaluate the following:

${}^{8}\mathrm{P}_{3}$

Ans. : We have,

${}^{8}\mathrm{P}_{3} = \frac{8!}{(8-3)!}\left[\because\ {}^{\mathrm{n}}\mathrm{P}_{\mathrm{r}} = \frac{\mathrm{n}!}{(\mathrm{n}-\mathrm{r})!}\right]$

$= \frac{8\times7\times6\times5}{5!}$

$= 336$

Hence, ${}^{8}\mathrm{P}_{3} = 336$

*** Given section consists of questions of 3 marks each.**

133. If the different permutations of all the letter of the word EXAMINATION are listed as in a dictionary, how many words are there in this list before the first word starting with E?

Ans. : In this problem, we have to find the number of words starting with E. Here in EXAMINATION we have two I's and two N's and all other letters are different.

$\therefore$ Number of ways of arrangement $= \frac{10!}{2!2!}$

$= \frac{10\times9\times8\times7\times6\times5\times4\times3\times2!}{2\times1\times2!}$= 907200

134. In an examination a question paper consist of 12 questions divided into two parts i.e. part I and part II containing 5 and 7 questions, respectively. A student is required to attempt 8 questions in all, selecting at least 3 from each part. In how many ways can a student select the questions?

Ans. : Here number of questions in part I are 5 and number of questions in part II are 7. We have to select 8 questions at least 3 questions from each section. So we have required selections are 3 from part I and 5 from part II or 4 from part I and 4 from part II or 5 from part I and 3 from part II.

$\therefore$ Number of ways of selection $= {}^5C_3 \times {}^7C_5 + {}^5C_4 \times {}^7C_4 + {}^5C_5 \times {}^7C_3$

$= \frac{5!}{3!2!} \times \frac{7!}{5!2!} + \frac{5!}{4!1!} \times \frac{7!}{4!3!} + \frac{5!}{5!0!} \times \frac{7!}{5!0!}$

$= \frac{5\times4\times3!}{3!\times2\times1} \times \frac{7\times6\times5!}{5!\times2\times} + \frac{5\times4!}{4!\times1} \times \frac{7\times6\times5\times4!}{4!\times3\times2\times1} + 1 \times \frac{7\times6\times5\times4!}{3\times2\times1\times4!}$

$= 10 \times 21 + 5 \times 35 + 1 \times 35$

= 210 + 175 + 35 = 420

135. From a class of 25 students, 10 are to be chosen for an excursion party. There are 3 students who decide that either all of them, will join or none of them will join. In how many ways can the excursion party be chosen?

Ans. : Here total students are 25 from which 10 are chosen for an excursion party. If the 3 students join the party then 7 students out of remaining 22 students or 3 students not join the party them 10 students out of remaining 22 students.

$\therefore$ Number of ways of selection $= {}^3C_3 \times {}^{22}C_7 + {}^3C_0 \times {}^{22}C_{10}$

$= 1 \times \frac{22!}{7!15!} + 1 \times \frac{22!}{10!12!}$

$= \frac{22\times21\times20\times19\times18\times17\times16\times15!}{7\times6\times5\times4\times3\times2\times1\times15!} + \frac{22\times21\times20\times19\times18\times17\times16}{10\times9\times8\times7\times6\times5\times4\times3}$

= 170544 + 646646 = 817190

136. Find: r, if 5 ${}^4P_r$ = 6 ${}^5P_{r-1}$

Ans. : We have, 5. ${}^4P_r$ = 6 . (${}^5P_{r-1}$)

$\Rightarrow 5 \cdot \frac{4!}{(4-r)!} = 6 \times \frac{5!}{[5-(r-1)]!}$

$\Rightarrow \frac{5\cdot4!}{(4-r)!} = \frac{6\times5\times4!}{(6-r)!}$

$\Rightarrow \quad \frac{1}{(4-r)!} = \frac{6}{(6-r)(5-r)(4-r)!}$

$\Rightarrow$ (6 - r) (5 - r) = 6

$\Rightarrow$ 30 - 11r + r^2 = 6

$\Rightarrow$ r^2 - 11r + 24 = 0

$\Rightarrow$ (r - 3) (r - 8) = 0

$\Rightarrow$ r = 3, 8

But r $\neq$ 8, because in ^4P_r, r cannot be greater than 4.

Hence, r = 3

137. How many words, with or without meaning, each of 3 vowels and 2 consonants can be formed from the letters of the word INVOLUTE?

Ans. : We have, in the word INVOLUTE, there are 4 vowels, namely, I, O, E, U and 4 consonants, namely, N, V, L and T.

The number of ways of selecting 3 vowels out of 4 = $^4C_3$ = 4.

The number of ways of selecting 2 consonants out of 4 = $^4C_2$ = 6. Thus, the number of combinations of 3 vowels and 2 consonants is 4 $\times$ 6 = 24.

Now, we each of these 24 combinations has 5 letters which can be arranged among themselves in 5 ! ways. Thus, the required number of different words is 24 $\times$ 5 ! = 2880

138. In how many ways can the letters of the word ASSASSINATION be arranged so that all the S's are together?

Ans. : There are 13 letters in the word 'ASSASSINATION' out of which 3 are A's, 4 are S's, 2 are I's, 2 are N's and the rest are all distinct.

Considering all S's together and treating them as one letter we have 10 letters.

These 10 letters can be arranged in $= \frac{10!}{3!\,2!\,2!}$

$= \frac{10\times9\times8\times7\times6\times5\times4\times3!}{3!\times2\times2}$

$= 10\times9\times8\times7\times6\times5$

$= 151200$

Hence, the total words are 151200.

139. prove that:

$$\frac{n!}{(n-r)!r!}+\frac{n!}{(n-r+1)!(r-1)!}=\frac{(n+1)!}{r(n-r+1)!}$$

Ans. : We have,

$$\text{L.H.S.} = \frac{n!}{(n-r)!r!}+\frac{n!}{(n-r+1)!(r-1)!}$$

$$= \frac{n!}{(n-r)!r\times\left[(r-1)!\right]}+\frac{n!}{(n-r+1)\left[(n-r)!\right](r-1)!}$$

$$= \frac{n!}{(n-r)!\times(r-1)!}\left[\frac{1}{r}+\frac{1}{n-r+1}\right]$$

$$= \frac{n!}{(n-r)!\times(r-1)!}\left[\frac{n-r+1+r}{r(n-r+1)}\right]$$

$= \frac{n!}{(n - r)! \times (r-1)!}\left[\frac{n+1}{r(n-r+1)}\right]$

$= \frac{(n+1) \times n!}{(n-r+1)! \times (n-r)! \times r \times (r-1)!}$

$= \frac{(n+1!)}{(n-r+1)! \times r!}$

$= \frac{(n+1)!}{r!(n-r+1)!}$

R.H.S

$\therefore$ L.H.S.= R.H.S.

140. How many 3-digit even number can be made using the digits 1, 2, 3, 4, 5, 6, 7, if no digits is repeated?

Ans. : In order to find the number of even digits, we fix the unit's digit as an even digit.

Fixing the unit's digit as 2:

Number of arrangements possible = ${}^6P_2 = 6 \times 5 = 30$

Similarly, fixing the unit's digit as 4:

Number of arrangements possible = ${}^6P_2 = 6 \times 5 = 30$

Fixing the unit's digit as 6:

Number of arrangements possible = ${}^6P_2 = 6 \times 5 = 30$

$\therefore$ Number of 3-digit even numbers that can be formed = 30 + 30 + 30 = 90

141. How many natural numbers not exceeding 4321 can be formed with the digits 1, 2, 3 and 4, if the digits can repeat?

Ans. : The given digits are 1, 2, 3 and 4. These digits can be repeated while forming the numbers. So, number of required four digit natural numbers can be found as follows.

Consider four digit natural numbers whose digit at thousandths place is 1.

Here, hundredths place can be filled in 4 ways. (Using the digits 1 or 2 or 3 or 4)

Similarly, tens place can be filled in 4 ways. (Using the digits 1 or 2 or 3 or 4)

Ones place can be filled in 4 ways. (Using the digits 1 or 2 or 3 or 4)

Number of four digit natural numbers whose digit at thousandths place is 1 = $4 \times 4 \times 4 = 64$

Similarly, number of four digit natural numbers whose digit at thousandths place is 2 = $4 \times 4 \times 4 = 64$

Now, consider four digit natural numbers whose digit at thousandths place is 4:

Here, if the digit at hundredths place is 1, then tens place can be filled in 4 ways and ones place can also be filled in 4 ways.

If the digit at hundredths place is 2, then tens place can be filled in 4 ways and ones place can also be filled in 4 ways.

If the digit at hundredths place is 3 and the digit at tens place is 1, then ones place can be filled in 4 ways.

If the digit at hundredths place is 3 and the digit at tens place is 2, then ones place can be filled only in 1 way so that the number formed is not exceeding 4321.

Number of four digit natural numbers not exceeding 4321 and digit at thousandths place is 3 = 4 × 4 + 4 × 4 + 4 + 1 = 37

Thus, required number of four digit natural numbers not exceeding 4321 is 64 + 64 + 64 + 37 = 229.

142. How many words can be formed with the letters of the word 'PARALLEL' so that all L's do not come together?

Ans. : There are 8 letters in the word 'PARALLEL' out of which A's and 3 are L's and the rest are all distinct.

So, total number of words

$= \frac{8!}{3!\,2!}$

$= \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3!}{2 \times 1 \times 3!}$

$= 8 \times 7 \times 6 \times 5 \times 2$

$= 3360.$

Considering all L's together and treating them as one letter we have 6 letters out of which A repeats 2 times and others are distinct. These 6 letters can be arranged in $\frac{6!}{2!}$ ways.

So, the number of words in which all L's come together $= \frac{6!}{2!}$

$= \frac{6 \times 5 \times 4 \times 3 \times 2!}{2!}$

$= 6 \times 5 \times 4 \times 3 \times 2$

$= 360$

Hence, the number of words in which all L's do not come together

= 3360 - 360

= 3000

143. Find the total number of ways in which six '+' and four '−' signs can be arranged in a line such that no two '−' signs occur together.

Ans. : Total number of '+' signs = 6

Total number of '-' signs = 4

six '+' signs can be arranged in a row in $= \frac{6!}{6!} = 1$ Way [∵ All '+' signs are identical]

Now, we are left with seven places in which four different things can be arranged in $^7P_4$ ways but all the four '-' signs are identical, therefore, four "-' signs can be arranged

$= \frac{^7\mathrm{P}_4}{4!}$

$= \frac{7!}{\frac{(7-4)!}{4!}} = \frac{7!}{3! \times 4!}$

$= \frac{7\times6\times5\times4!}{3\times2\times4!} = 7 \times 5 = 35$

Hence, the required number of ways = 1 × 35 = 35.

144. In how many ways can three jobs I, II and III be assigned to three persons A, B and C if one person is assigned only one job and all are capable of doing each job?

Ans. : Total number of jobs = 3

$\therefore$ the number of ways to assined these job is to

three persons = 3 × 2 × 1

= 6

145. m men and n women are to be seated in a row so that no two women sit together, if m > n then show that the number of ways in which they can be seated as $\frac{m!(m+1)!(m-n+1)!}{m!(m+1)!(m-n+1)!}$

Ans. : m men can be seated in a row in ${}^{m}p_{m}$ = m! ways.

Now, in the (m + 1) gaps n women can be arranged in ${}^{m+1}p_{n}$ ways.

Hence, the number of ways in which no two women sit together.

$= m! \times {}^{m+1}P_{n}$

$= m! \times \frac{(m+1)}{(m+1-n)!}$

$= m! \times \frac{(m+1)}{(m-n+1)!}$

Hence, proved.

146. How many different words can be formed with the letters of word 'SUNDAY'? How many of the words begin with N? How many begin with N and end in Y?

Ans. : There are 6 letters in the word 'SUNDAY'. The total number of words formed with these 6 letters is the number of arrangements of 6 items, taken all at a time, which is equal to ${}^{6}P_{6}$ = 6!

= 6 × 5 × 4 × 3 × 2 × 1

= 720

If we fix up Nin the begining, then the remaining 5 letters can be arranged in ${}^{5}P_{5}$ = 5! ways so, the total num bar of words which begin which N = 5!

= 5 × 4 × 3 × 2 × 1

= 120

if we fix up Nin the begining and Y at the end, then the remaining 4 letters can be arranged in ${}^{4}P_{4}$ = 4! ways.

So, the total number of words which begin with N and end with Y = 4! = 4 × 3 × 2 × 1 = 24.

147. How many words can be formed from the letters of the word 'SERIES' which start with S and end with S?

Ans. : In the word 'SERIES' there are 6 letters of which 2 are S and 2 are E's.
Let us fix 5 at the extreme left and at the extreme right end. Now, we are left Let us fix 5 at the extreme left and at the extreme right end. Now, we are left $\frac{4!}{2!}$ ways.
Hence, required number of arrangements $= \frac{4!}{2!}$
$= \frac{4\times3\times2!}{2!}$
$= 12$

148. In how many ways can the letters of the word 'ARRANGE' be arranged so that the two R's are never together?

Ans. : There are 7 letters in the word 'ARRANGE' out of which 2 are A's 2 are R's and the rest are all distinct.
So, total number of words $= \frac{7!}{2!\,2!}$
$= \frac{7\times6\times5\times4\times3\times2!}{2\times2!}$
$= 7\times6\times5\times2\times3$
$= 1260$
Considering all R's together and treating them as one letter we have 6 letters out of which A repeats 6! 2 times and other are distinct. These 6 letters can be arranged in $\frac{6!}{2!}$ ways.
So, the number of words in which all R's come together $= \frac{6!}{2!}$
$= \frac{6\times5\times4\times3\times2!}{2!} = 360$
Hence, the number of words in which all R's do not came together,
= Total number of words - Number of words in which all R's come together
= 1260 - 360
= 900.

149. If $\frac{(2n!)}{3!(2n-3)!}$ and $\frac{n!}{2!(2n-2)!}$ are in the ratio 44 : 3 find n.

Ans. : We have,

$\frac{\frac{(2n!)}{3!(2n-3)!}}{\frac{n!}{2!(n-2)!}} = \frac{44}{3}$

$\Rightarrow \frac{(2n)!\times2!(n\text{-}2)!}{3!(2n-3)!\times n!} = \frac{44}{3}$

$\Rightarrow \frac{(2n)(2n-1)(2n\text{-}2)(2n\text{-}3)!\times2!(n-2)!}{3\times2(2n\text{-}3)\times n(n-1)(n\text{-}2)} = \frac{44}{3}$

$\Rightarrow \frac{2n(2n\text{-}1)\times2(n\text{-}1)}{3(n\text{-}1)} = \frac{44}{3}$

$\Rightarrow 4(2n-1) = 44$

$\Rightarrow 2n-1 = 11$

$\Rightarrow 2n = 12$

$\Rightarrow n = 6$

$\therefore n = 6$

150. Find the number of diagonals of:

ii. A hexagon.
iii. A polygon of 16 sides.

Ans. :

i. A hexagon:

A hexagon has 6 angular points. By joining any two angular points we get a line which is either a side or a diagonal.

Number of line $= {}^{6}\mathrm{C}_{2} = \frac{6!}{2!4!}$

$= \frac{6\times 5}{2} = 15$

Number of sides = 6

Number of diagonals = 15 - 6 = 9

ii. A polygon of 16 sides:

A polygonof 16 sides will have 16 angular points. By joining any 2 points we get a line which is either a side or a diagonal.

Number of lines $= {}^{16}\mathrm{C}_{2} = \frac{16!}{2!14!}$

$= \frac{16\times 15}{2} = 120$

Number of sides = 16

Number of diagonals = 120 - 16 = 104

151. A committee of 7 has to be formed from 9 boys and 4 girls. In how many ways can this be done when the committee consists of:

i. Exactly 3 girls?
ii. At least 3 girls?
iii. At most 3 girls?

Ans. :

i. The committee consists of exaclty 3 girls.

we have to select 4 boys from 9 boys.

This can be done in ways nd 3 girls out of 4 girls can br selected in ways.

The required number ways $= {}^{9}\mathrm{C}_{4} \times {}^{4}\mathrm{C}_{3}$

$= \frac{9\times 8\times 7\times 6}{4\times 3\times 2\times 1} \times 4$

$= 504$

ii. At least 3 girls are there.

There are 3 or more than 3 or 4 girls.

a. 3 girls and 4 boys $= {}^{4}\mathrm{C}_{3} \times {}^{9}\mathrm{C}_{3}$

b. 4 girls and 3 boys $= {}^{4}\mathrm{C}_{4} \times {}^{9}\mathrm{C}_{3}$

The required number ways $= {}^{4}\mathrm{C}_{3} \times {}^{9}\mathrm{C}_{4} + {}^{4}\mathrm{C}_{4} \times {}^{9}\mathrm{C}_{7}$

$= 504 + 84$

$= 588$

iii. For at most 3 girls theres are 3, 2, 1.

a. 0 girls and 7 boys $= {}^{4}\mathrm{C}_{0} \times {}^{9}\mathrm{C}_{7}$

b. 1 girls and 7 boys $= {}^{4}\mathrm{C}_{1} \times {}^{9}\mathrm{C}_{6}$

c. 2 girls and 7 boys $= {}^4C_2 \times {}^9C_5$

d. 3 girls and 7 boys $= {}^4C_3 \times {}^9C_4$

Total number of required ways

$= {}^4C_0 \times {}^9C_7 + {}^4C_1 \times {}^9C_6 + {}^9C_2 \times {}^9C_5 + {}^4C_3 \times {}^9C_4$

$= 0 \times \frac{9\times8}{2} + 4 \times \frac{9\times8\times7}{3\times2} + \frac{4\times3}{2} \times \frac{9\times8\times7\times6}{4\times3\times2} + 504$

$= 36 + 48 \times 7 + 18 \times 42 + 504$

$= 1630$

152. If ${}^{15}C_{3r} = {}^{15}C_{r+3}$, Find r.

Ans. : We have,

If ${}^nC_p = {}^nC_q = n$

Then p + q = n

Also,

${}^{15}C_{3r} = {}^{15}C_{r+3}$,

⇒ 3r + r + 3 = 15

4r + 3 = 15

4r = 15 - 3

4r = 12

r = 3

153. If ${}^{15}C_r : {}^{15}C_{r\text{-}1}, = 11 : 5,$ Find r.

Ans. : We have,

$\frac{{}^{15}C_r}{{}^{15}C_{r-1}} = \frac{11}{5}$

$\Rightarrow \frac{15-r+1}{r} = \frac{11}{5}$

$\Rightarrow 75 - 5r + 5 = 11r$

$\Rightarrow\ 16r = 80$

$\Rightarrow r = 5$

154. There are 10 points in a plane of which 4 are collinear. How many different straight lines can be drawn by joining these points.

Ans. : Number of point = 10

Number of collinear points = 4

Since 4 out of 10 points are collinear, so the number of liner will be,

Number of liner ${}^{10}C_2 = \left({}^4C_2 - 1\right)$

$= {}^{10}C_2 - {}^4C_2 + 1$

$= \frac{10!}{2!8!} - \frac{4!}{2!2!} + 1$

$= \frac{10\times9}{2} - \frac{4\times3}{2} + 1$

$= 45 - 6 + 1$

$= 40$

155. A business man hosts a dinner to 21 guests. He is having 2 round tables which can accommodate 15 and 6 persons each. In how many ways can he arrange the guests?

Ans. : In One round table the business man can accommodate the guests in ways. In the second round table he can the guests in ways. Keeping one guest as fixed in the round table, the other 14 guests can be arrange in 14! ways. Keeping one guest as fixed in the second round tabie, the other 5 guests can be number of ways in which the guests can be arrange is $= {}^{21}C_{15} \times {}^{6}C_{6} \times 14! \times 5!\text{ ways}$

156. For all positive integers n, show that ${}^{2n}C_n + {}^{2n}C_{n-1} = \frac{1}{2}\left({}^{2n+2}C_{n+1}\right)$.

Ans. : We have,

$\text{L.H.S.} = {}^{2n}C_n + {}^{2n}C_{n-1}$

$= \frac{2n!}{n!n!} + \frac{2n!}{(n-1)!(n-1)!}$

$= (2n)!\left[\frac{1}{n(n-1)!(n)(n-1)!} + \frac{1}{(n-1)!(n-1)!}\right]$

$= \frac{(2n!)}{(n-1)!(n-1)!}\left[\frac{1+n^2}{n^2}\right] \ldots(i)$

${}^{2n+2}C_{n+1} = \frac{(2n+2)!}{(n+1!)(n+1)!}$

$= \frac{(2n+2)(2n+1)(2n!)}{n(n+1)n(n-1)!} \ldots(ii)$

$= \frac{(2n!)}{(n-1)!(n-1)!} \times \frac{(n+1)^2(n^2)(n-1)!(n-1)!}{(2n-2)(2n+1)(2n!)} \times \left(\frac{1+n^2}{n^2}\right)$

$= \frac{(n+1)(n^2+1)}{(2n+1)} \times \frac{1}{2}$

157. If ${}^{2n}C_3 : {}^{n}C_2 = 44 : 3,$ find n.

Ans. : We have,

$\Rightarrow \frac{\frac{2n!}{(3n)!(2n-3!)}}{\frac{n!}{2!(n-2!)}}$

$\Rightarrow \frac{2n!2!(n-2)!}{3!(2n-3)!n!} = \frac{44}{3}$

$\Rightarrow \frac{2n!}{3n\ (n-1)!(2n-3)!} = \frac{44}{3}$

$\Rightarrow 2n(2n-1)(2n-2) = 44n(n-1)$

$\Rightarrow (2n-1)(n-1) = 11(n-1)$

$n = 6$

158. In a certain city, all telephone numbers have six digits, the first two digits always being 41 or 42 or 46 or 62 or 64. How many telephone numbers have all six digits distinct?

Ans. : If first two digits is 41, then the remaining 4 digits can be arranged in ${}^{8}P_{4}$ ways

$= \frac{8!}{(8-4)} = \frac{8\times7\times6\times5\times4!}{4!} = 1680$

Similarly firth two digits can be 42 or 46 or 62 or 64

$\therefore$ Total number of telephone number have all digits distinct = 5 × 1680 = 8400

Hence, the required telephone numbers = 8400

159. A sports team of 11 students is to be constituted, choosing at least 5 from Class XI and atleast 5 from Class XII. If there are 20 students in each of these classes, in how many ways can the team be constituted?

Ans. : Total number of students in each class = 20

We have to select at least 5 students from each class.

So we can select either 5 students from class XI and 6 students from class XII or 6 students from class XI and 5 students from class XII.

$\therefore$ Total number of ways of selecting a team of 11 players $= {}^{20}C_5 \times {}^{20}C_6 + {}^{20}C_6 \times {}^{20}C_5 = 2 \times {}^{20}C_5 \times {}^{20}C_6$

160. A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has.
   i. No girls.
   ii. At least one boy and one girl.
   iii. At least three girls.

Ans. : Number of girls = 4

Number of boys = 7

We have to select a team of 5 members provided that

i. Team having no. girls

$\therefore$ Required number of ways $= {}^{7}\mathrm{C}_5 = \frac{7\times6}{2!} = 21$

ii. Team having at least one boy and one girl

$\therefore$ Required number of ways

$= {}^{7}\mathrm{C}_1 \times {}^{4}\mathrm{C}_4 + {}^{7}\mathrm{C}_2 \times {}^{4}\mathrm{C}_3 + {}^{7}\mathrm{C}_3 \times {}^{4}\mathrm{C}_2 + {}^{7}\mathrm{C}_4 \times {}^{4}\mathrm{C}_1$

$= 7 \times 1 + 21 \times 4 + 35 \times 6 + 35 \times 4$

$= 7 + 84 + 210 + 140$

$= 441$

iii. Team having at least three girls

Required number of ways

$= {}^{4}\mathrm{C}_3 \times {}^{7}\mathrm{C}_2 + {}^{4}\mathrm{C}_4 \times {}^{7}\mathrm{C}_1$

$= 4 \times 21 + 7 = 84 + 7 = 91$

161. If ${}^{n}C_{r-1} = 36$, ${}^{n}C_{r} = 84$ and ${}^{n}C_{r+1} = 126$, then find ${}^{r}C_2$.

[**Hint:** Form equation using $\frac{{}^{\mathrm{n}}\mathrm{C}_{\mathrm{r}}}{{}^{\mathrm{n}}\mathrm{C}_{r+1}}$ and $\frac{{}^{\mathrm{n}}\mathrm{C}_{\mathrm{r}}}{{}^{\mathrm{n}}\mathrm{C}_{r-1}}$ to find the value of r.]

Ans. : We know that $\frac{{}^{\mathrm{n}}\mathrm{C}_{\mathrm{r}}}{{}^{\mathrm{n}}\mathrm{C}_{\mathrm{r}}} = \frac{\mathrm{n-r+1}}{\mathrm{r}}$

$\therefore \frac{\mathrm{n-r+1}}{\mathrm{r}} = \frac{84}{36}$ (given)

$\Rightarrow \frac{\mathrm{n-r+1}}{\mathrm{r}} = \frac{7}{3}$

$\Rightarrow 3_{\mathrm{n}} - 3\mathrm{r} + 3 = 7\mathrm{r}$

$\Rightarrow\ 10r - 3n = 3 \ldots.(i)$

$\frac{^nC_{r+1}}{^nC_r} = \frac{n-(r+1)+1}{r+1} = \frac{126}{84}$

$\therefore\ \frac{n-r}{r+1} = \frac{3}{2}$

$\Rightarrow\ 2n - 2r = 3r + 3$

$\Rightarrow\ 2n - 5r = 3 \ldots.(ii)$

Solving (i) and (ii), we get n = 9 and r = 3.

$\therefore\ {}^{r}2_2 = {}^{3}C_2 = 3$

162. How many automobile license plates can be made if each plate contains two different letters followed by three different digits?

Ans. : There are 26 English alphabets and 10 digits (0 to 9).
It is given that each plate contains two different letters followed by three different digits.

$\therefore$ Arrangment of 26 letters, taken 2 at a time $= {}^{26}P_2 = 26 \times 25 = 650$ And arrangment of 10 digits, takan three at a time $= {}^{10}P_3 = 10 \times 9 \times 8 = 720$

$\therefore$ Total number of licence plates $= 650 \times 720 = 468000$

*** Given section consists of questions of 5 marks each.** **[80]**

163. Find the number of words formed by permuting all the letters of the following words:
INDEPENDENCE.

Ans. : There are 12 letters in the word 'INDEPENDENCE' out of which 2 are D'S, 3 are N'S, 4 are E'S and the rest are all distinct.

so, the total number of words $= \frac{12!}{2!\,3!\,4!}$

$= \frac{12\times11\times10\times9\times8\times7\times6\times5\times4!}{2!\,3!\,4!}$

$= \frac{12\times11\times10\times9\times8\times7\times6\times5}{2\times3\times2}$

$= 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5$

$= 1663200.$

164. Find the number of words formed by permuting all the letters of the following words:
EXERCISES.

Ans. : There are 9 letters in the word 'EXERCISES' out of which 3 are E's, 2 are S's and the rest are all distinct.
So, the total number of words

$= \frac{9!}{3!\,2!}$

$= \frac{9\times8\times7\times6\times5\times4\times3!}{3!\times2\times1}$

$= 9 \times 8 \times 7 \times 6 \times 5 \times 2$

$= 30240$

165. Find the number of words formed by permuting all the letters of the following words:
INDIA.

Ans. : There are 5 letters in the word 'INDIA' out of which 2 are I'S, and the rest are all distinct.
so, the total number of
$= \frac{5!}{2!}$
$= \frac{5\times4\times3\times2!}{2!}$
$= 60$

166. The letters of the word 'ZENITH' are written in all possible orders. How many words are possible if all these words are written out as in a dictionary? What is the rank of the word 'ZENITH'?

Ans. : In a dictionary the words at each stage are arranged in alphabetical order. In the given problem we must therefore consider the words beginning with E, H, I, N, T, Z in order. 'E' will occur in the first place as often as there are ways of arranging the remaining 5 letters all at a time i.e. E will occur S! times. Similarly H will occur in the first place the same number of times.
$\therefore$ Number of words starting with E = 5! = 5 × 4 × 3 × 2 × 1 = 120
Number of words starting with H = 5! = 120
Number of words starting with I = 5! = 120
Number of words starting with N = 5! = 120
Number of words starting with T = 5! = 120
Number of words beginning with Z is S!, but one of these words is the word ZEN ITH i tselr. So, we first find the number of words beginning with ZEH, ZEI and ZENH,
Number of words starting with ZEH = 3! = 6
Number of words starting with ZEI = 3! = 6
Number of words starting with ZENH = 2! = 2.
Now, the words beginning with ZENI must follow.
There are 21 words beginning with ZENI one of these words is the word ZENITH itself. The first word beginning with ZENI is the word ZENI HT and the next word is ZENITH.
$\therefore$ Rank of ZENITH
= 5 × 120 + 2 × 6 + 2 + 2
= 600 + 12 + 4
= 600 + 16
= 616

167. In how many ways can the letters of the word "INTERMEDIATE" be arranged so that:
i. The vowels always occupy even places?

ii. The relative order of vowels and consonants do not alter?

Ans. : INTERMEDIATE

I = 2 times, T = 2 times, E = 3 times, N, R, M, D, A

Number of letters = 12

i. There are 6 vowels. They ocuepy even places 2nd, 4th, 6th, 8th, 10th, 12th. After there six there are six places and 5 letters, T is 2 times. So, number of ways for consonants $= \frac{6!}{2!}$

The total number of ways when vowels ocuepy even places

$= \frac{6!}{2!} \times \frac{6!}{2!\,3!}$

$= \frac{6\times5\times4\times3\times2\times6\times5\times4\times3\times2}{2\times2\times3\times2}$

$= 21600$

Required number of ways= 21600

ii. Number of ways such that relative order of vowels and consonants do not alter

$= \frac{6!}{2!} \times \frac{6!}{2!\,3!}$

$= 21600$

Required number of ways= 21600

168. In how many ways can 4 red, 3 yellow and 2 green discs be arranged in a row if the discs of the same colour are indistinguishable?

Ans. : 4 red, 3 yellow and 2 green discs.

Total discs = 9

Required number of ways

$= \frac{9!}{4!\,3!\,2!}$

$= \frac{9\times8\times7\times6\times5}{3\times2\times2}$

$= 1260$

Required number of ways = 1260.

169. A bag contains 5 black and 6 red balls. Determine the number of ways in which 2 black and 3 red balls can be selected.

Ans. : We have,

Bag contains 5 black and 6 red balls.

Number of ways to select 2 black balls out of 5 black and 3 red balls out of 6 red balls.

$= {}^5C_2 \times {}^6C_3$

$= \frac{5\times4}{2} \times \frac{6\times5\times4}{3\times2}$

$= 200$

170. How many words can be formed by taking 4 letters at a time from the letters of the word 'MORADABAD'?

Ans. : The given word is 'MORADABAD'.

Number of M = 1, Number of O = 1

Number of R = 1, Number of A = 3

Number of D = 2, Number of B = 1

i. Number of arrangement of 4 letters.

Selected from these $= {}^6\mathrm{C}_4 \times 4!$

$= 15 \times 24$

$= 360$

ii. Two ailke and with more than one

So, one pair from these and 2 from letters from rest 5 letters.

Number of ways to arrange therefour

$= {}^2\mathrm{C}_1 \times {}^5\mathrm{C}_2 \times \frac{4!}{2!}$

$= 2 \times 10 \times 12$

$= 240$

iii. Two ailke and with more than

Number of ways to arrange therefour

$= {}^2\mathrm{C}_2 \times {}^5\mathrm{C}_2 \times \frac{4!}{2!2!}$

$= 6$

iv. There alike and one different number of ways to the therefour

$= 1 \times {}^5\mathrm{C}_1$

$= 5 \times \frac{4!}{3!1!}$

$= 20$

Required number of ways = 240 + 360 + 6 + 20

Required number of ways = 626

171. A parallelogram is cut by two sets of m lines parallel to its sides. Find the number of parallelograms thus formed.

Ans. : In a parallelogram, there are 2 sets of parallel line. Each set of parallel lines consists of lines and each parallelogram is formed by choosing two line from the first and two straing lines the second set.

Hence, the total nmber of parallelogram $= {}^{\mathrm{m}+2}\mathrm{C}_2 \times {}^{\mathrm{m}+2}\mathrm{C}_2$

$= ({}^{\mathrm{m}+2}\mathrm{C}_2)^2$

172. A committee of 3 persons is to be constituted from a group of 2 men and 3 women. In how many ways can this be done? How many of these committees would consist of 1 man and 2 women?

Ans. : We have,

The number of ways selecting of 3 people out of 5

$\Rightarrow {}^5\mathrm{C}_3 = \frac{5!}{3!2!}$

$= \frac{5\times4}{2} = 10$

1 man can be selected from 2 men in ways and 2 wimen can be selected fron 3 ways,

The required number of committees

$= {}^{2}C_1 \times {}^{3}C_2$

$= \frac{2!}{1!1!} \times \frac{3!}{2!1!}$

$= 6$

173. In how many ways can one select a circket team of eleven from17 players in which only 5 persons can bowl if each cricket team of 11 must include exactly 4 bowlers?

Ans. : We have,

There are total 5 bowlwrs and 12 batsm are avilable to select from.

Number of ways to select a team and 11 that includes exactly 4 bowlers.

= (7 batsm out of 12 bastsman) and (4 bowlers out of 5 bowlers)

$= {}^{12}C_7 \times {}^{5}C_4$

$\Rightarrow \frac{12\times11\times10\times9\times8}{5\times4\times3\times2\times1} \times 5$

$\Rightarrow 3960$

174. Evaluate ${}^{20}C_5 + \sum_{r=2}^{5} {}^{25-x}C_4$.

Ans. : We have,

$\Rightarrow {}^{20}C_5 + \sum_{r=2}^{5} {}^{25-x}C_4$

$\Rightarrow \left({}^{20}C_5 + {}^{20}C_4\right) + {}^{21}C_4 + {}^{22}C_4 + {}^{23}C_4$

$\Rightarrow \left({}^{21}C_5 + {}^{21}C_4\right) + {}^{22}C_4 + {}^{23}C_4$

$\Rightarrow {}^{23}C_5 + {}^{23}C_4$

$\Rightarrow {}^{24}C_5$

$\Rightarrow 42504$

175. From 4 officers and 8 jawans in how many ways can 6 be chosen:

i. To include exactly one officer.

ii. To include at least one officer?

Ans. : Total number of officer = 4

Total number of jawans = 8

Total number of selection to be made = 6

i. To include exactly one officer.

This can be done is ${}^{4}C_1 \times {}^{8}C_5$

$= \frac{4!}{1!3!} \times \frac{8!}{5!3!}$

$= \frac{4\times8\times7\times6}{3\times2} = 224$

ii. To include at least one officer

This can be done is

$$ {}^4C_1 \times {}^8C_5 + {}^4C_2 \times {}^8C_4 + {}^4C_1 \times {}^8C_3 + {}^4C_4 \times {}^8C_2 $$

$$ = \frac{4\times 8!}{5!3!} + \frac{4!}{2!2!} + \frac{8!}{4!4!} + \frac{4!}{3!1!} \times \frac{8!}{3!5!} + \frac{1\times 8!}{2!6!} $$

$$ = \left(\frac{4\times8\times7\times6}{3\times2}\right) + \left(\frac{4\times3\times8\times7\times6\times5}{2\times4\times3\times2}\right) + \left(\frac{4\times8\times7\times6}{3\times2}\right) + \left(\frac{8\times7}{2\times1}\right) $$

$$ = (4\times8\times7) + (4\times3\times7\times5) + (4\times8\times7) + (4\times7) $$

$$ = 224 + 420 + 224 + 28 $$

$$ = 896 $$

176. If $\alpha = {}^mC_2$, then find the value of ${}^{\alpha}C_2$.

Ans. : We have,

$$ \alpha = {}^mC_2 = \frac{m(m-1)}{2} $$

$$ {}^{\alpha}C_2 = \frac{\alpha(\alpha-1)}{2} $$

$$ = \frac{\left(\frac{m(m-1)}{2}\right)\left(\frac{m(m-1)}{2}-1\right)}{2} $$

$$ = \frac{m(m-1)(m^2-m-2)}{2\times2\times2} $$

$$ = \frac{m(m-1)(m+1)(m-2)}{8} $$

$$ = \frac{m(m-1)(m+1)(m-2)}{4\times2} $$

Multiplying with 3, numerator and denominator to make 4.

$$ = \frac{m(m+1)m(m-1)(m-2)}{4.3.2.1} $$

$$ = \frac{3(m+1)m(m-1)(m-2)}{4!} $$

$$ = 3.{}^{m+1}C_4 $$

177. Find the number of positive integers greater than 6000 and less than 7000 which are divisible by 5, provided that no digit is to be repeated.

Ans. : We have to form 4-digit numbers which are greater than 6000 and less than 7000.

We know that a number is divisible by 5, if at the unit place of the number there is 0 or 5.

So, unit digit can be filled in 2 ways.

The thousandth place can be filled by '6' only.

The hundredth place and tenth place can be filled together in 8 × 7 = 56 ways. So, total number of ways = 56 × 2 = 112

178. Match each item given under the column C_1 to its correct answer given under the column C_2.

Five boys and five girls form a line. Find the number of ways of making the seating arrangement under the following condition:

	C_1		C_2
(a)	Boys and girls alternate.	(i)	5! × 6!

(b)	No two girls sit together.	(ii)	10! – 5! 6!
(c)	All the girls sit together.	(iii)	$(5!)^2 + (5!)^2$
(d)	All the girls are never together.	(iv)	2! 5! 5!

Ans. :

	C_1		C_2
(a)	Boys and girls alternate.	(iii)	$(5!)^2 + (5!)^2$
(b)	No two girls sit together.	(i)	5! × 6!
(c)	All the girls sit together.	(iv)	2! 5! 5!
(d)	All the girls are never together.	(ii)	10! – 5! 6!

Explanation:

Total number of arrangment when boys and girls alternate: = $(5!)^2 + (5!)^2$

1. No two girls sit together: = 5! 6!
2. All the girls sit never toghether = 2! 5! 5!
3. All the girls sit never together = 10! - 5! 6!

*** Case study based questions** **[8]**

179. Five students Ajay, Shyam, Yojana, Rahul and Akansha are sitting in a playground in a line.

Based on the above information, answer the following questions.

(i) Total number of ways of sitting arrangement of five students is
(a) 120 (b) 60 (c) 24 (d) None of these

(ii) Total number of arrangement of sitting, if Ajay and Yojana sit together, is
(a) 60 (b) 48 (c) 72 (d) 120

(iii) Total number of arrangement 'Yojana and Rahul sitting at extreme position' is
(a) 24 (b) 36 (c) 48 (d) 12

(iv) Total number of arrangement, if shyam is sitting in the middle, is
(a) 24 (b) 12 (c) 6 (d) 36

(v) Total number of arrangement sitting Yojana and Rahul not sit together, is

(a) 72 (b) 120 (c) 60 (d) 144

Ans. : (i) (a) We have five students.

$\therefore$ Total number of arrangements is $\mathbf{5! = 120}$

(ii) (b) Ajay and Yojana sit together.

Total number of arrangements $= 4! \times 2! = 24 \times 2 = 48$

(iii) (d) Total number of arrangements of Yojana and Rahul sitting in extreme position is $2! \times 3! = \; 2 \times 6 = 12$

(iv) (a) Number of arrangements of Shyam in middle is

$4! = 24$

(v) (a) Number of arrangement Yojana and Rahul not sit together, is

$\frac{4!}{2!} \times 3! = 72 \text{ or } 5! - 4! \times 2! = 72$

180. Republic day is a national holiday of India. It honours the date on which the constitution of India came into effect on 26 January 1950 replacing the Government of India Act (1935) as the governing document of India and thus, turning the nation into a newly formed republic.

Answer the following question, which are based on the word "REPUBLIC".

(i) Find the number of arrangements of the letters of the word 'REPUBLIC'.

(a) 40300 (b) 30420 (c) 40320 (d) 40400

(ii) How many arrangements start with a vowel?

(a) 12015 (b) 15120 (c) 12018 (d) 15100

(iii) Which concept is used for finding the arrangements start with a vowel?

(a) Permutation (b) FPM (c) Combination (d) FPA

(iv) If the number of arrangements of the letters of the word 'REPUBLIC' is abcde, the (a + b + $\mathbf{c} + \mathbf{d} + \mathbf{e})$ is

(a) 10 (b) 9 (c) 8 (d) 15

(v) If the number of arrangements start with a vowel is abcde, then $(\mathbf{a} + \mathbf{b}) - (\mathbf{d} + \mathbf{e})$ is

(a) 2 (b) 3 (c) 4 (d) 5

Ans. : (i) (c) The letters in the word 'REPUBLIC' are all distinct. There are 8 letters in the given word. So, the number of arrangements are 8 ! i.e. 40320.

(ii) (b) The vowels in a given word are ' $\mathrm{E}, \mathrm{I}, \mathrm{U}$ '. If we start a word from vowel, we can choose 1 vowel from 3 vowels in ${}^{3}\mathbf{C}_{1}$ ways. Further, remaining 7 letters can be arranged in 7 ! ways.

$\therefore$ Total number of arrangements start with a vowel

$= {}^{3}\mathrm{C}_{1} \times 7! = 3 \times 5040 = 15120$

(iii) (c) Combination

(iv) (b) Since, number of arrangements are 40320 .

On comparing, we get

$$a=4, b=0, c=3, d=2, e=0$$

$$\text{So, } a+b+c+d+e=4+0+3+2+0=9$$

(v) (c) Since, number of arrangements are 15120

On comparing, we get

$$a=1, b=5, c=1, d=2, e=0$$

$$\therefore (a+b)-(d+e)=(1+5)-(2+0)=6-2=4$$

----- When we strive to become better than we are, everything around us becomes better too -----